Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

"NP means not polynomial": false, it means checkable in polynomial time

"every problem in P is in NP": true

"if 3-colouring is in P then P = NP": true, it is NP-complete

2.2

$9 - 5 = \mathbf{4}$

the vertices outside a cover have no edge between them, and the reverse is also true

2.3

check each vertex has one of 3 colours, and each edge joins two different colours

$O(V + E)$: polynomial, so 3-colouring is in NP

2.4

$2^{60} \approx 1.15 \times 10^{18}$ subsets

1.15×10^{9} s

about **37 years**

3.1

(a) TEAM is in NP (answers can be checked fast); a known NP-complete problem reduces *to* TEAM

(b) given (G, k), build the complement $\bar{G}$ in $O(V^2)$; a set is a clique in G exactly when it has no edges in $\bar{G}$; so the answers are the same, and the reduction is polynomial

(c) every NP problem could be solved in polynomial time: P = NP

3.2

(a) the chosen edges share no vertex (a matching); any cover must contain at least one end of each chosen edge, so OPT $\geq$ the number of chosen edges; the algorithm takes exactly 2 per chosen edge, so at most 2 OPT

(b) edges 1–2, 3–4, 5–6: cover of 6 vertices; the optimum is $\{2, 4, 5\}$, 3 vertices, so exactly twice

3.3

(a) a vertex per exam, an edge for each shared student; give each vertex one of 10 colours so that no edge has the same colour at both ends; graph colouring, which is NP-complete

(b) 10^{40}: far too many to check, even at 10^{12} a second

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(c) order exams by number of conflicts, largest first, and give each the first slot that works; the schedule is always valid, but it may need more slots than the best one

(d) greedy first; if it needs more than 10 slots, use an integer-programming or SAT solver, which handles 40 exams in practice: real instances are small and structured, so exact solvers often finish quickly despite the worst case