## Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

**2.1**

$V - 1$ rounds of relaxing every edge

if one more round still lowers a distance, there is a negative cycle

**2.2**

$1000^3 = \mathbf{10^9}$ steps

when all pairs are needed on a dense graph, or edges may be negative

**2.3**

edges leaving the S side: S→A 10, B→D 9

capacity **19**

every flow is at most 19; if a flow of 19 is found, both are optimal

**2.4**

add S with an edge of capacity 1 to each student, and an edge of capacity 1 from each project to T

each allowed pair is an edge of capacity 1 from student to project

the maximum flow's full student→project edges form a maximum matching

**3.1**

(a) e.g. S–A–D–T 8; S–A–C–T 2; S–B–D–T 2 and S–B–D–C–T 6; S–B–D–A–C–T 1, using the residual edge D→A

(b) **19**

(c) S side $\{S, B\}$: edges S→A 10 and B→D 9; capacity 19 = the flow, so both are optimal

**3.2**

(a) A 2, B 5; C 5, T 7

(b) Dijkstra settles A at 4 (smaller than B's 5) and then C at 7, T at 9; B later offers A = 2, but A is already settled, so T stays 9 instead of 7

(c) no: there is no cycle at all, since every edge leads forward towards T

**3.3**

(a) S → each exam (1); exam → allowed room (1); room → T (1); the maximum flow is the largest number of exams placed

(b) e.g. E1–R2, E2–R1, E4–R3, E5–R4: 4 exams; E3 is left out

(c) E1, E2 and E3 can only use R1 and R2: 3 exams, 2 rooms, so Hall's condition fails



(d) make R3, R4 or R5 able to take one of E1–E3 (e.g. move equipment), or run E3 at another time: adding one such edge raises the matching to 5



exercises.lol exercises.com.cn