Solutions

2.1

- $C(5) = C(4) + C(3) + 1$ with $C(0) = C(1) = 1$: $C(5) = 15$ ✓.

2.2

- $dp[5] = 2$ (4+1) ✓.

2.3

- The recurrence, the base cases, the fill (dependency) order ✓.

3.1

- $dp[0] = 0$; $dp[1] = 1$ (coin 1); $dp[2] = 2$ (1+1); $dp[3] = 1$ (coin 3) ✓.
- $dp[4] = 1$ (coin 4); $dp[5] = 2$ (4+1); $dp[6] = 2$ (3+3) ✓.
- $dp[6] = 2$ with coins 3, 3 ✓.
- Greedy pays 3 (4+1+1) – the table is correct; no exchange argument exists for this coin system ✓.

3.2

- After (1,1): $dp = [0, 1, 1, 1, 1, 1, 1, 1]$ ✓.
- After (3,4): $dp = [0, 1, 1, 4, 5, 5, 5, 5]$ – taking both items at capacity 4 gives value $1 + 4 = 5$; after (4,5): $dp[7] = 4 + 5 = 9$ ✓.
- Final: $dp[7] = 9$ – items (3,4) and (4,5): weight 7, value 9 ✓.
- Downward iteration prevents reusing an item within the same pass (upward would allow the item twice – that is unbounded knapsack) ✓.
- Naive recursion: $\Theta(2^n)$ (include/exclude per item); the table: $\Theta(n \times \text{capacity})$ ✓.

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