Solutions

2.1

- Four – (1,4), (5,7), (8,11), (12,16) ✓.

2.2

- 5 and 9 (merge to 14) ✓.

2.3

- Any optimal solution can swap its picks for greedy's, one at a time, staying feasible and the same size ✓.

3.1

- Picks: (1,4), then (5,7), (8,11), (12,16) ✓.
- First rejections: (3,5) overlaps (1,4); (0,6) overlaps (1,4) ✓.
- Exchange: if the optimal starts with activity $O_1 \neq (1,4)$, then finish(1,4) $\leq$ finish(O_1), so replacing O_1 by (1,4) leaves every later activity feasible ✓.
- Repeat down the list: the optimal schedule transforms into the greedy one, unchanged in size ✓.
- Greedy size 4 = optimal size 4 ✓.

3.2

- Merges: 5+9 = 14; 12+13 = 25; 14+16 = 30; 25+30 = 55; 45+55 = 100 ✓.
- Weighted depth: $\sum f_i d_i = 224$ bits ✓.
- Fixed length: $\lceil \log_2 6 \rceil = 3$ bits $\times 100 = 300$ ✓.
- Leaf-swap argument: in an optimal tree, the two smallest frequencies sit deepest; if not, swapping them with deeper leaves cannot increase the cost ✓.
- So the greedy merge (two smallest first) is exactly what optimality permits – induction on the merge count closes it ✓.

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