Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

`[4, -1, 2, 1]`

sum **6**

2.2

take 1 from the right while 3 left items wait: +3

take 6 while 8 waits: +1

4 split inversions

2.3

after each partition it recurses into one side only

with random pivots the sizes shrink by a constant factor on average: $n + \frac{3}{4}n + \cdots = O(n)$

2.4

$3^{13} = 3^{12} \cdot 3 = (3^6)^2 \cdot 3$

$\equiv 1 \cdot 3 = \mathbf{3}$

5 multiplications: 3 squarings and 2 products

3.1

(a) $n(n-1)/2 \approx \mathbf{5 \times 10^9}$; $n \log_2 n \approx \mathbf{1.7 \times 10^6}$

(b) sort by x and split at the middle line; find the closest pair in each half, distance δ; then check pairs that cross the line, only inside the strip of width 2δ

(c) points on each side are at least δ apart, so only a few (at most 7) can lie in a $\delta \times 2\delta$ box above a point in the strip, sorted by y

(d) $T(n) = 2T(n/2) + O(n)$, so $O(n \log n)$

3.2

(a) best suffix of left: 5; best prefix of right: $-1 + 2 = 1$, so crossing **6**

(b) left: 5; right: 4

(c) **6**: `[5, -1, 2]`

3.3

(a) sort: $10^9 \times 30 = \mathbf{3 \times 10^{10}}$; heap: each score is compared with the smallest kept, and at most $\log_2 100 \approx 7$ steps to replace it: at most about $\mathbf{7 \times 10^9}$, usually far fewer

(b) sorting stores all $10^9 \times 8$ B $= 8$ GB; the heap stores 100 scores, 800 B

(c) it needs all the data in memory and several passes over it

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(d) a min-heap of the best 100: one pass, tiny memory, and a new score is rejected after one comparison when it is below the heap's minimum