Solutions

2.1

- The merge: $\Theta(n)$ per level ✓.

2.2

- It splits, recurses (once), and combines in $O(1)$ – one branch is still the pattern ✓.

2.3

- A full pass over the half on each side: $\Theta(n)$ ✓.

3.1

- Split at the midpoint: left $[-2, 1, -3, 4]$, right $[-1, 2, 1, -5, 4]$ ✓.
- Left best: 4 (the single element). Right best: $2 + 1 = 3$ ✓.
- Crossing: best left suffix 4, best right prefix $2{+}1 = 3$, crossing total $4{+}(-1){+}2{+}1 = 6$ – winner 6, subarray $[4, -1, 2, 1]$ ✓.
- Recurrence $2T(n/2) + \Theta(n) = \Theta(n \log n)$ – the crossing scan is a full pass, so the per-level work stays linear ✓.

3.2

- Balanced expectation: $2 \times 64 \times \ln 64 \approx 2 \times 64 \times 4.159 \approx 532$ ✓.
- Measured first-element on the course array: 356 – the array is not adversarial ✓.
- Sorted worst: $\sum_{k=1}^{64}(k-1) = 64 \times 63/2 = 2016$ comparisons; the earlier figure 2080 counts the final pass's extra compares – quote 2016 for the sum formula, 2080 measured ✓.
- Worst-case recurrence: $T(n) = T(n-1) + \Theta(n)$ – each partition peels exactly one element ✓.
- Randomised pivots make every input behave like a random one in expectation – the worst case remains possible but no longer adversarially triggerable ✓.

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