Solutions

2.1

- Balanced: $n^{\log_3 3} = n = f(n)$ – case 2 ✓. $T(n) = \Theta(n \log n)$.

2.2

- $\log_4 n$ levels of constant work: $\Theta(\log n)$ ✓.

2.3

- 2^i nodes of $n/2^i$: the level product is n at every level ✓.

3.1

- $9T(n/3) + n$: leaves $n^{\log_3 9} = n^2$ dominate the linear root – $T(n) = \Theta(n^2)$ ✓.
- $T(2n/3) + 1$: leaf total is $n^0 = 1$ (a = 1); the root work 1 matches – case 2 with a shrinking ratio: $T(n) = \Theta(\log n)$ (base 3/2) ✓.
- $3T(n/4) + n \log n$: leaves $n^{\log_4 3} \approx n^{0.79}$; the root $n \log n$ dominates (polylog factor included) – $T(n) = \Theta(n \log n)$ ✓.

3.2

- Hypothesis: $T(m) \leq c\, m \log_2 m$ for all $m < n$ ✓.
- Substitute: $T(n) \leq 2c\frac{n}{2} \log_2 \frac{n}{2} + n = cn \log_2 n - cn + n$ ✓.
- Residual $n(1 - c) \leq 0$ exactly when $c \geq 1$ – the induction closes ✓.
- The measurement (310 under 321) does not prove the bound – it confirms the constant is realistic on real data: theory bounds, measurement witnesses ✓.

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