Exercise walk-through

2.4
Counting and combinatorics

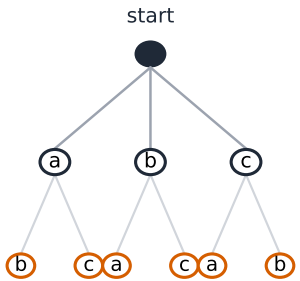
University Computer Science

Counting and combinatorics: two rules, two formulas

1. **Rules** Disjoint alternatives ADD; sequential choices MULTIPLY - 4 routes $\times$ 3 vehicles $= 12$.
2. **Formulas** $P(n,r) = n!/(n-r)!$ ordered; $C(n,r) = n!/(r!(n-r)!)$ unordered - the $r!$ is the whole difference.
3. **IE** $|A \cup B \cup C|$ = singles $-$ pairs $+$ triple: $45+35+30-12-10-8+5 = 85$; team of 3 from 8 with a leader: 168 both ways.

Counting trees: ordered vs unordered

the tree: ordered vs unordered



ordered pairs (ab ≠ ba): $3 \times 2 = 6$
unordered pairs {a,b} = {b,a}: $6 \div 2! = 3$

sequential choices MULTIPLY; disjoint alternatives ADD

the two formulas

$P(n, r) = n!/(n-r)!$ ordered
$C(n, r) = n!/(r!(n-r)!)$ unordered

same choices; combinations divide by the r! arrangements of each set

inclusion–exclusion, three sets

$|A \cup B \cup C| = |A|+|B|+|C| -$ pairs + triple
$45+35+30 - 12-10-8 + 5 = 85$; none: 15
add singles, subtract pairs, add the triple

Branches multiply, alternatives add, overcounts subtract.

Exercises 2.1·2.3

2.1 Committee of 3 from 9 – how many? [3]

2.2 Arrange 5 distinct books on a shelf – how many orders? [3]

2.3 $|A| = 12$, $|B| = 9$, $|A \cap B| = 4$ – $|A \cup B|$? [3]

Exercises 2.1·2.3

2.1 Committee of 3 from 9 – how many? [3]

2.1

- $\binom{9}{3} = 84$ ✓.

2.2 Arrange 5 distinct books on a shelf – how many orders? [3]

2.2

- $5! = 120$ ✓.

2.3 $|A| = 12$, $|B| = 9$, $|A \cap B| = 4$ – $|A \cup B|$? [3]

2.3

- $12 + 9 - 4 = 17$ ✓.

Exercise 3.1

3.1 A password is 4 characters from 26 letters and 10 digits, with at least one digit required (repetition allowed).

Count without restriction (36^4), subtract all-letter passwords (26^4) – the complement method ✓. Compute both powers and the final answer ($36^4 - 26^4 = 1679616 - 456976 = 1222640$) ✓. Then count the version with NO repetition allowed ($P(36,4) - P(26,4) = 1054920$) ✓. [8]

Exercise 3.1

3.1 A password is 4 characters from 26 letters and 10 digits, with at least one digit required (repetition allowed).

Count without restriction (36^4), subtract all-letter passwords (26^4) – the complement method ✓. Compute both powers and the final answer ($36^4 - 26^4 = 1679616 - 456976 = 1222640$) ✓. Then count the version with NO repetition allowed ($P(36,4) - P(26,4) = 1054920$) ✓. [8]

3.1

- Without restriction: each of 4 slots picks from 36 – 36^4 ✓.
- All-letter: 26^4; at least one digit: subtract – complement ✓.
- $36^4 = 1679616$; $26^4 = 456976$; difference 1222640 ✓.
- No repetition: $36 \cdot 35 \cdot 34 \cdot 33 = 1413720$; all-letter no repetition $26 \cdot 25 \cdot 24 \cdot 23 = 358800$; answer 1054920? – recompute: $1413720 - 358800 = 1054920$ ✓.

Exercise 3.2

3.2 Among 60 students: 30 read fiction, 25 non-fiction, 20 poetry; 12 fiction and non-fiction, 9 fiction and poetry, 7 non-fiction and poetry; 4 all three.

Compute at-least-one by inclusion-exclusion ($30 + 25 + 20 - 12 - 9 - 7 + 4 = 51$) and exactly-two categories ($12 - 4 + 9 - 4 + 7 - 4 = 16$) ✓. Verify the partition: exactly-one $= 51 - 16 - 4 = 31$ and none $= 60 - 51 = 9$ – the four numbers sum to 60 ✓. [7]

Exercise 3.2

3.2 Among 60 students: 30 read fiction, 25 non-fiction, 20 poetry; 12 fiction and non-fiction, 9 fiction and poetry, 7 non-fiction and poetry; 4 all three.

Compute at-least-one by inclusion-exclusion ($30 + 25 + 20 - 12 - 9 - 7 + 4 = 51$) and exactly-two categories ($12 - 4 + 9 - 4 + 7 - 4 = 16$) ✓. Verify the partition: exactly-one $= 51 - 16 - 4 = 31$ and none $= 60 - 51 = 9$ – the four numbers sum to 60 ✓. [7]

3.2

- At least one: $30 + 25 + 20 - 12 - 9 - 7 + 4 = 51$ ✓.
- Exactly two: each pair count minus the triple, summed: $(12-4)+(9-4)+(7-4) = 16$ ✓.
- Exactly one: $51 - 16 - 4 = 31$; none: 9 ✓.
- Partition check: $31 + 16 + 4 + 9 = 60$ ✓.