

Exercise walk-through

2.2

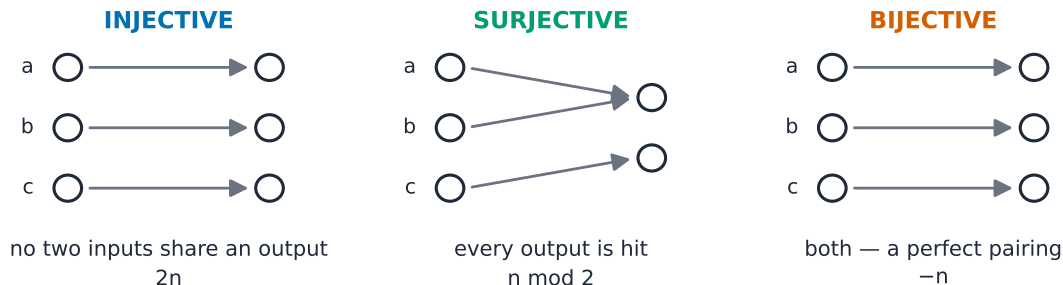
Sets, functions and relations

University Computer Science

Sets, functions and relations: arrows and matrices

- 1 **Sets** Union, intersection, difference, complement; the **power set** of A pairs in/out bits - one bit each.
- 2 **Functions** $f(n) = 2^n$ **injective** not **surjective**; $\lfloor n/2 \rfloor$ surjective not injective; $-n$ bijective (own inverse).
- 3 **Relations** Reflexive = diagonal 1s; symmetric = matrix = transpose; transitive = check length-2 paths.

Set operations, arrow diagrams and relation properties



set algebra

$A \cup B$, $A \cap B$, $A - B$, complement
power set pairs subsets with bit strings
composition $g \circ f$ reads right-to-left

relation properties (matrix view)

reflexive: diagonal all 1
symmetric: matrix = its transpose
transitive: check paths of length 2

Arrows classify the function; the matrix reads the relation at a glance.

Exercises 2.1·2.3

2.1 How many subsets does a 4-element set have? [3]

2.2 Is $f(n) = n + 1$ on the integers injective, surjective, both? [3]

2.3 What does a symmetric relation's matrix look like? [3]

Exercises 2.1·2.3

2.1 How many subsets does a 4-element set have? [3]

2.1

- $2^4 = 16$ ✓.

2.2 Is $f(n) = n + 1$ on the integers injective, surjective, both? [3]

2.2

- Bijective – shifts never merge inputs and every integer is hit ✓.

2.3 What does a symmetric relation's matrix look like? [3]

2.3

- Symmetric about the diagonal: matrix = its transpose ✓.

Exercise 3.1

3.1 $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6\}$.

Compute $|A \cup B|$, $|A \cap B|$, $|A - B|$, $|B - A|$ ✓ and verify $|A \cup B| = |A| + |B| - |A \cap B|$ numerically ✓. Then count the functions from A to B ($3^4 = 81$) and the injections from B to A ($4 \times 3 \times 2 = 24$), naming the rule used each time ✓. [8]

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3.1

- $A \cup B = \{1, 2, 3, 4, 6\}$: 5; $A \cap B = \{2, 4\}$: 2 ✓.
- $A - B = \{1, 3\}$: 2; $B - A = \{6\}$: 1 ✓.
- Check: $5 = 4 + 3 - 2$ ✓ – inclusion-exclusion holds.
- Functions $A \rightarrow B$: each of 4 inputs picks one of 3 outputs – $3^4 = 81$ (multiplication rule) ✓.
- Injections $B \rightarrow A$: 4 choices, then 3, then 2 – 24 ✓.

Exercise 3.2

3.2 Three relations on $\{a, b, c\}$, given as sets of pairs.

$R_1 = \{(a, a), (b, b), (c, c)\}$: reflexive, symmetric, antisymmetric, transitive ✓. $R_2 = \{(a, b), (b, a)\}$: symmetric not reflexive; is it transitive? – no: $(a, b), (b, a)$ demand (a, a) , absent ✓. R_3 with $(a, b), (b, c)$ only: transitive requires (a, c) – test what is missing ✓. State which of the three is an equivalence relation (only R_1) and why ✓. [7]

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3.2

- R_1 : all four properties hold (identity relation) ✓.
- R_2 : symmetric, not reflexive (no loops); transitivity fails on the round trip without (a, a) ✓.
- R_3 : transitive requires the composite (a, c) – its absence decides ✓.
- Equivalence = reflexive + symmetric + transitive: only R_1 qualifies ✓.