

Exercise walk-through

2.1

Logic and proofs

University Computer Science

Logic and proofs: translate, then choose the shape

- 1 **Equivalences** $p \rightarrow q \equiv \neg p \vee q$; $p \rightarrow q \equiv \neg q \rightarrow \neg p$; $\neg(p \wedge q) \equiv \neg p \vee \neg q$;
negation swaps $\forall$ with $\exists$.
- 2 **Shapes** Direct (assume p , derive q) · **contraposition** (prove $\neg q \rightarrow \neg p$) ·
contradiction (assume $p \wedge \neg q$, absurdity).
- 3 **Named** n^2 even $\rightarrow$ n even, by contraposition: $n = 2k+1$ gives $n^2 = 2(2k^2+2k)+1$, odd.

A truth table and its proof tree

the table checks

p	q	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

matching columns prove:
 $p \rightarrow q \equiv \neg q \rightarrow \neg p$ (contraposition)

DIRECT

assume p, derive q

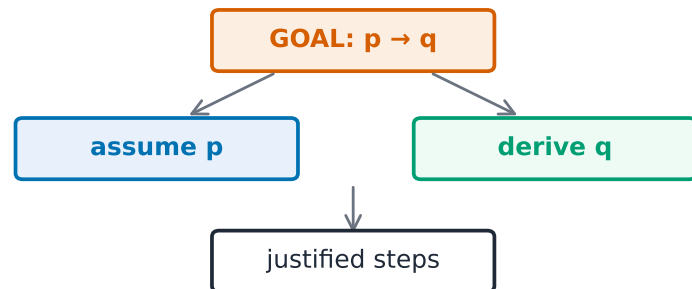
CONTRAPOSITION

prove $\neg q \rightarrow \neg p$

CONTRADICTION

assume $p \wedge \neg q$, absurdity

the tree structures



The table checks; the tree structures — three proof shapes choose your data.

Exercises 2.1-2.3

2.1 State the contrapositive of $p \rightarrow q$.

[3]

2.2 Negate $\forall x P(x)$ and $\exists x P(x)$.

[3]

2.3 Apply De Morgan to $\neg(r \vee s)$.

[3]

Exercises 2.1-2.3

2.1 State the contrapositive of $p \rightarrow q$.

[3]

2.1

- $\neg q \rightarrow \neg p$ ✓.

2.2 Negate $\forall x P(x)$ and $\exists x P(x)$.

[3]

2.2

- $\exists x \neg P(x); \forall x \neg P(x)$ ✓.

2.3 Apply De Morgan to $\neg(r \vee s)$.

[3]

2.3

- $\neg r \wedge \neg s$ ✓.

Exercise 3.1

3.1 Three claims to prove, one per shape.

(i) If $3n + 2$ is odd then n is odd – name the shape, prove it ✓. (ii) $\sqrt{2}$ is irrational – name the shape (contradiction), write the parity case analysis on $p^2 = 2q^2$ ✓. (iii) If $xy = 0$ then $x = 0$ or $y = 0$ – CONTRAPOSITION of the conclusion: show $x \neq 0$ and $y \neq 0$ forces $xy \neq 0$ ✓. For each: state the strategy BEFORE the proof, as the course rule demands. [8]

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3.1

- (i) Contraposition: n even $\Rightarrow n = 2k \Rightarrow 3n + 2 = 6k + 2$ even ✓.
- (ii) Contradiction: $\sqrt{2} = p/q$ lowest terms $\Rightarrow p^2 = 2q^2 \Rightarrow p$ even $\Rightarrow q$ even $\Rightarrow$ terms not lowest – absurd ✓.
- (iii) Contrapositive of the conclusion: $x \neq 0$ and $y \neq 0 \Rightarrow xy \neq 0$ – negates the premise directly ✓.
- Strategy named before each proof, per the course rule ✓.

Exercise 3.2

3.2 An argument is presented: the premises are $p \rightarrow q$ and p ; the conclusion is q – and a second one with premises $p \rightarrow q$ and q , concluding p .

Verify the first with a two-row truth table (modus ponens: rows where both premises hold force q) ✓. Show the second is INVALID – exhibit the row $p = F$, $q = T$ where premises hold and conclusion fails (that shape is affirming the consequent) ✓. [7]

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3.2

- Modus ponens table: only row $p = T, q = T$ satisfies both premises; $q = T$ there ✓.
- Second: row $p = F, q = T$ satisfies $p \rightarrow q$ and q but p fails – invalid ✓.
- The invalid shape is affirming the consequent; a truth-table row is the proof of invalidity ✓.