

Exercise walk-through

13.3

Probability models

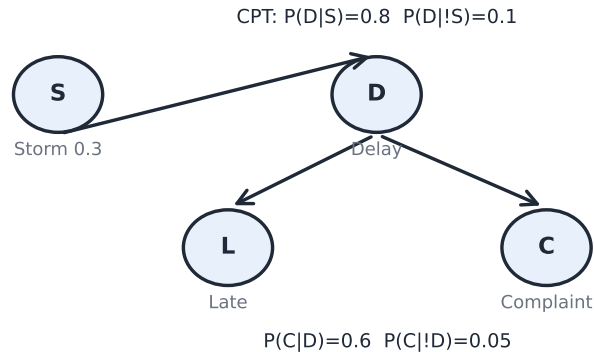
University Computer Science

Probability models: evidence updates belief

- 1 Numbers** $P(S) 0.3$, $P(D|S) 0.8$, $P(D|\text{not}S) 0.1$, $P(C|D) 0.6$, $P(C|\text{not}D) 0.05$:
 $P(C|S) 0.49$, $P(C|\text{not}S) 0.105$, $P(C) 0.2205$.
- 2 Posterior** $P(S|C) = 0.49 \times 0.3 / 0.2205 = 0.667$ - a complaint multiplies storm belief 2.5-fold; the no-storm leak is not negligible.
- 3 Rare** With $P(S) 0.03$: posterior 0.126 - same evidence, belief collapsing to one-eighth; the normaliser and base rate carry the arithmetic.

Network graph

evidence flows up the arrows



the inference, in numbers:

$$\begin{aligned}
 P(C|S) &= 0.8 \times 0.6 + 0.2 \times 0.05 = 0.49 \\
 P(C|\neg S) &= 0.1 \times 0.6 + 0.9 \times 0.05 = 0.105 \\
 P(C) &= 0.49 \times 0.3 + 0.105 \times 0.7 \\
 &= 0.2205 \\
 P(S|C) &= 0.49 \times 0.3 / 0.2205 \\
 &= 0.667
 \end{aligned}$$

rare storms ($P=0.03$): posterior 0.126

the base rate is not a footnote

query $P(\text{Storm} \mid \text{Complaint})$: evidence explains backwards through the leak

Storm raises Delay; Delay explains Late and Complaint.

Exercises 2.1-2.3

2.1 What does the network factorise?

[3]

2.2 Compute $P(D)$.

[3]

2.3 Name the two fallacies this unit warns about.

[3]

Exercises 2.1-2.3

2.1 What does the network factorise? [3]

2.1

- The joint distribution, into per-node CPTs given parents ✓.
-

2.2 Compute $P(D)$. [3]

2.2

- $P(D) = 0.8 \times 0.3 + 0.1 \times 0.7 = 0.24 + 0.07 = 0.31$ ✓.
-

2.3 Name the two fallacies this unit warns about. [3]

2.3

- Dropping the normaliser; swapping $P(C | S)$ with $P(S | C)$ (the prosecutor's fallacy) ✓.

Exercise 3.1

3.1 Using the course net, compute $P(D \mid C)$.

Enumerate: $P(C \mid D)P(D)$ and $P(C \mid \neg D)P(\neg D)$; give $P(C)$ and then $P(D \mid C)$. Give $P(S \mid C)$ from the same normaliser and state which cause the complaint implicates more. Explain in one line why the two posteriors sum to more than

1.

[8]

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[8]

3.1

- $P(C \mid D)P(D) = 0.6 \times 0.31 = 0.186 \checkmark$.
- $P(C \mid \neg D)P(\neg D) = 0.05 \times 0.69 = 0.0345$; $P(C) = 0.2205 \checkmark$.
- $P(D \mid C) = 0.186/0.2205 = \mathbf{0.844} \checkmark$.
- $P(S \mid C) = 0.49 \times 0.3/0.2205 = 0.667$ – the complaint implicates Delay more than Storm $\checkmark$.
- They do not sum to 1: S and D are not mutually exclusive exhaustive answers – D hides inside both S-branches $\checkmark$.

Exercise 3.2

3.2 The storm prior drops to 0.03 (rare storms).

Recompute $P(C)$ and $P(S \mid C)$. State which changed most and why. Give the screening-test reading of the same arithmetic (a rare disease, a positive test) and the design lesson for tests used at scale.

[7]

Exercise 3.2

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Recompute $P(C)$ and $P(S | C)$. State which changed most and why. Give the screening-test reading of the same arithmetic (a rare disease, a positive test) and the design lesson for tests used at scale. [7]

3.2

- $P(C) = 0.49 \times 0.03 + 0.105 \times 0.97 = 0.11655$ ✓.
- $P(S | C) = 0.0147/0.11655 = \mathbf{0.126}$ (from 0.667) ✓.
- Changed most: $P(S | C)$ – the rare cause loses the most belief, collapsing to one-eighth ✓.
- Screening reading: a rare disease with a decent test still yields most positives being false at low prevalence – design for high specificity or test twice ✓.