

Exercise walk-through

11.1

Automata and regular languages

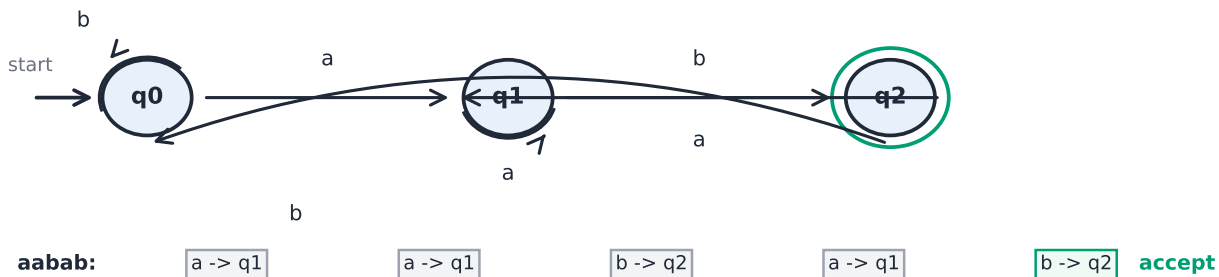
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Automata: the table decides

- 1 **Trace** ends-in-ab DFA on aabab: $q_0 \ q_1 \ q_1 \ q_2 \ q_1 \ q_2$ - accept; aba ends q_1 - reject; the suite asserts every step.
- 2 **Pump** Any long accepted string splits xyz with pumpable y - $a^n b^n$ is not regular: pumping the a-block breaks the count.
- 3 **Guess** NFAs branch; subset construction tracks the branches at once - determinisation can square the state count.

State diagrams and acceptance trace

a DFA that accepts strings ending in ab



pumping ceiling: no finite machine matches $a^n b^n$ — pump the loop and the count breaks

Three states; the trace of aabab accepts at q_2 .

Exercises 2.1-2.3

2.1 In the ends-in-ab DFA, which state accepts? [3]

2.2 Why is aba rejected? [3]

2.3 One line: what does the pumping lemma forbid a regular language from doing? [3]

Exercises 2.1-2.3

2.1 In the ends-in-ab DFA, which state accepts? [3]

2.1

- q_2 ✓.

2.2 Why is aba rejected? [3]

2.2

- It ends in state q_1 – the final **a** leaves the pattern incomplete ✓.

2.3 One line: what does the pumping lemma forbid a regular language from doing? [3]

2.3

- It forbids unbounded counting: every long string has a pumpable loop ✓.

Exercise 3.1

3.1 The ends-in-ab DFA over the alphabet a, b has these transitions: q_0 on a goes to q_1 ; q_1 on a stays q_1 ; q_1 on b goes to q_2 ; q_2 on a goes to q_1 ; q_0 on b stays q_0 ; q_2 on b goes to q_0 . The accepting state is q_2 .

Give the full state trace of **babbab** and the verdict. Give the trace of **babba**. State which two positions of the input could be swapped to flip the verdict, and why. Draw (or describe) the missing 8th transition a complete table needs if any input letter were added. [8]

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3.1

- **babbab**: $q_0 \rightarrow q_0$ (b), $q_0 \rightarrow q_1$ (a), $q_1 \rightarrow q_2$ (b), $q_2 \rightarrow q_0$ (b), $q_0 \rightarrow q_1$ (a), $q_1 \rightarrow q_2$ (b) – accepted ✓.
- **babba**: q_0 , q_0 , q_1 , q_2 , q_1 , q_1 – rejected ✓.
- Swapping the last two letters flips it: **babbab** keeps ... ab ; **babba**... variants end ba or aa – only strings ending ab accept, so the final pair decides ✓.
- A complete table over $\{a, b, c\}$ needs a dead state q_d : q_d loops on every letter, entered on c from anywhere ✓.

Exercise 3.2

3.2 Use the pumping lemma to argue $L = \{a^n b^n\}$ is not regular with p as the pumping length.

Choose the string and state its constraint on n . Split it xyz and state what y must contain. Pump once and derive the contradiction. State what this ceiling means for a lexer that must match nested brackets. [7]

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3.2

- Choose $a^p b^p$ ($n = p$ forced by the lemma's threshold) ✓.
- $|xy| \leq p$ keeps y inside the a -block: $y = a^k$, $k \geq 1$ ✓.
- Pump once: $xy^2z = a^{p+k}b^p$ – more a 's than b 's, not in L ✓.
- Ceiling for lexers: nested brackets need counting, so no regex (DFA) lexer can match them – the parser above must take over ✓.