UNIVERSITY COMPUTER SCIENCE • EXERCISE SHEET

# 2.4 Counting and combinatorics

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

**KEY WORDS** permutation • combination

## Objective

Choose the rule; count two ways; run inclusion-exclusion.

## 1 Key knowledge and worked examples

### ■ The rules

Disjoint alternatives ADD; sequential choices MULTIPLY ✓. $P(n,r) = \frac{n!}{(n-r)!}$ ordered; $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$ unordered.

### ■ Worked example: inclusion-exclusion

Students: 45 tea, 35 coffee, 30 cocoa; pairs 12, 10, 8; triple 5. At least one: $45+35+30-12-10-8+5=85$; none: $100-85=15$ ✓.

## 2 Practice

**2.1** Committee of 3 from 9 – how many? [3]

**2.2** Arrange 5 distinct books on a shelf – how many orders? [3]

**2.3** $|A| = 12$, $|B| = 9$, $|A \cap B| = 4$ – $|A \cup B|$? [3]

## 3 Exam-style questions

**3.1** A password is 4 characters from 26 letters and 10 digits, with at least one digit required (repetition allowed).

Count without restriction ($36^4$), subtract all-letter passwords ($26^4$) – the complement method ✓. Compute both powers and the final answer ($36^4 - 26^4 = 1679616 - 456976 = 1222640$) ✓. Then count the version with NO repetition allowed ($P(36,4) - P(26,4) = 1054920$) ✓. [8]

**3.2** Among 60 students: 30 read fiction, 25 non-fiction, 20 poetry; 12 fiction and non-fiction, 9 fiction and poetry, 7 non-fiction and poetry; 4 all three.

Compute at-least-one by inclusion-exclusion ($30+25+20-12-9-7+4=51$) and exactly-two categories ($12-4+9-4+7-4=16$) ✓. Verify the partition: exactly-one $= 51-16-4=31$ and none $= 60-51=9$ – the four numbers sum to 60 ✓. [7]