11.3 Computability

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS Turing machine • halting problem

Objective

Run the diagonal argument and know what stays decidable.

1 Key knowledge and worked examples

■ The machine model

Tape, head, states – the formalisation of algorithm ✓. The halting problem: no decider exists; D flips H's answer about itself ✓.

■ Worked example: the decidable cousin

DFA emptiness: mark states reachable from q_0; empty iff no accepting state is marked – finiteness makes it searchable ✓.

2 Practice

2.1 State the halting problem in one sentence. [3]

2.2 What does $D(M)$ do in the proof? [3]

2.3 What does Rice's theorem cover? [3]

exercises.lol exercises.com.cn

3 Exam-style questions

3.1 Suppose $H(M, w)$ decides halting.

Write the construction of D from H precisely (both branches). Trace what happens when D runs on its own description. State the contradiction and what it proves about H. State the two mistakes in the claim "so no program ever stops" ✓. [8]

3.2 Three questions, three verdicts:

(1) Does DFA D accept any string at all? (2) Does TM M accept any string at all? (3) Does program P print banana on input x?

Give each verdict (decidable / undecidable) with a one-line reason. For the decidable one, give the algorithm. For (3), name the theorem and give the reduction's wrapper logic. [7]

exercises.lol exercises.com.cn