10.6 Reductions and NP-completeness

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS reduction • NP-completeness NP

Objective

Build reductions and read their direction correctly.

1 Key knowledge and worked examples

■ **The definition**

$A \leq_p B$: a polynomial translator from A-instances to B-instances – a B-solver solves A ✓. NP-complete = in NP + every NP problem reduces to it ✓.

■ **Worked example: the chain**

SAT → 3-SAT (clause splitting) → CLIQUE (triangle gadget) → VERTEX-COVER (complement) ✓.

2 Practice

2.1 Which direction carries hardness: $A \to B$ means – is hard? [3]

2.2 What two conditions define NP-completeness? [3]

2.3 Translate: G = triangle ABC + isolated D, $k = 3$. What VERTEX-COVER instance results? [3]

3 Exam-style questions

3.1 Reduce CLIQUE to VERTEX-COVER in full.

State the translator (graph, parameter) and its running time. Prove both directions: a k-clique exists in G iff a cover of size $n - k$ exists in $\bar{G}$. Run it on G = triangle ABC + isolated D with $k = 3$: list $\bar{G}$'s edges and the cover. State what a polynomial vertex-cover solver would then imply. [8]

3.2 A colleague proves their problem WARS hard by writing a SAT encoding of WARS and running a SAT solver.

Name the direction they actually proved (WARS $\leq_p$ SAT) and what it implies about a SAT solver. State what they must do instead to prove WARS NP-hard. State the two checks a full NP-completeness claim needs, and what remains unproven if the membership check is skipped. [7]