10.5 Graph optimisation

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS Kruskal • cuts

Objective

Run Kruskal by hand and know where Dijkstra breaks.

1 Key knowledge and worked examples

■ The safe edge

The lightest edge crossing any cut is in some MST – the property behind Kruskal (sort, accept, reject cycles) ✓.

■ Worked example: the course MST

Accept C–B 1, A–C 2, D–E 2, E–F 3, B–D 5; reject the cyclers. MST weight $1+2+2+3+5=13$ ✓.

2 Practice

2.1 In Kruskal's walk over the course graph, which is the first edge rejected, and why? [3]

2.2 What makes an edge light across a cut? [3]

2.3 Which algorithm handles negative-weight shortest paths? [3]

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3 Exam-style questions

3.1 Course edges: A–B 4, A–C 2, B–C 1, B–D 5, C–D 8, C–E 10, D–E 2, E–F 3, D–F 6.

Run Kruskal: sort the edges, walk them, accept or reject with a reason at each. Give the MST edges and total weight. Prove the first accepted edge C–B is safe using the cut argument (name the cut). [8]

3.2 Edges A–B 5, A–C 1, C–B 1 (all non-negative).

Run textbook Dijkstra (settle the nearest unsettled, never reconsider) from A: give the settle order and each node's final distance. State where the answer is wrong and why the light-edge argument does not protect it here. Give Bellman-Ford's complexity and what extra capability it buys. [7]

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