UNIVERSITY COMPUTER SCIENCE • EXERCISE SHEET

10.1 Recurrences and analysis tools

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS recurrence • recursion tree

Objective

Unroll recurrences, apply the master method, and close by substitution.

1 Key knowledge and worked examples

■ **The tree**

$T(n) = 2T(n/2) + \Theta(n)$: every level costs $\Theta(n)$, depth $\log_2 n$, so $T(n) = \Theta(n \log n)$ ✓.

■ **Worked example: the three master cases**

$2T(n/2)+n \to n \log n$ (balanced); $T(n/2)+1 \to \log n$ (leaves trivial); $4T(n/2)+n \to n^2$ (leaves n^2 dominate) ✓.

2 Practice

2.1 $T(n) = 3T(n/3) + n$: which master case, and what is the answer? [3]

2.2 $T(n) = T(n/4) + 1$: solve it. [3]

2.3 In the tree for $2T(n/2) + n$, what does level i cost? [3]

3 Exam-style questions

3.1 Solve each by the master method, naming the case:

$T(n) = 9T(n/3) + n$; then $T(n) = T(2n/3) + 1$; then $T(n) = 3T(n/4) + n \log n$.

For each, state the leaf total $n^{\log_b a}$ (or note the polynomial comparison), the dominating side, and the final class. Justify the middle case's comparison explicitly. [8]

3.2 Prove by substitution that $T(n) = 2T(n/2) + n$ satisfies $T(n) \le c\, n \log_2 n$ for some $c \ge 1$.

Write the induction hypothesis. Substitute it into the recurrence and simplify to show the residual is ≤ 0 exactly when $c \ge 1$. State what the measured count on the 64-element course array (310 against the 321 ceiling) adds to the proof. [7]