UNIVERSITY COMPUTER SCIENCE • EXERCISE SHEET

7.4 Virtual memory

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS page table • TLB

Objective

Translate addresses through the page table and price the faults.

1 Key knowledge and worked examples

■ **The machine**

16-bit virtual, 256-byte pages: 256 pages; 12-bit physical: 16 frames ✓. $0x0204$ = offset 4, VPN 2 → frame 5 → physical $0x0504$; $0x0210$ hits the TLB at $0x0510$ ✓.

■ **Worked example: the fault's price**

TLB hit 1 ns, walk 10 ns, fault 5 ms; a fault per million accesses amortises to 5 ns per access ✓.

2 Practice

2.1 20-bit virtual addresses with 1 KiB pages: how many virtual pages, and how many VPN bits? [3]

2.2 VPN 7 maps to frame 3. Translate $0x0705$ (256-byte pages). [3]

2.3 Distinguish an invalid access from a page fault in one line. [3]

3 Exam-style questions

3.1 A machine has 24-bit virtual addresses, 2 KiB pages, and 1 MiB of physical memory.

Compute the page count, the VPN bits, the frame count and the frame bits, and the page-table entry count. Translate virtual $0x123456$ (offset, VPN). Compute the page-table size in bits if each entry holds frame bits + valid + dirty + reference. [8]

3.2 A process's accesses hit the TLB 98% of the time; a miss walks the table in 20 ns; 1% of walks fault, each costing 4 ms.

Compute the average access time (TLB hit 1 ns). Recompute with faults at 0.1% of walks. State which term dominates each answer and the one-line design lesson. [7]

Solutions

2.1

- $2^{20}/2^{10} = 2^{10} = 1024$ pages; VPN = 10 bits ✓.

2.2

- Offset $0x05$; physical $= 3 \times 256 + 5 = 0x305$ ✓.

2.3

- Invalid = the page was never mapped (error); a fault = mapped but not resident (fetch it) ✓.

3.1

- Pages: $2^{24}/2^{11} = 2^{13} = 8192$ ✓.
- Frames: 1 MiB/2 KiB = 512 – frame bits 9; VPN bits $= 24 - 11 = 13$ ✓.
- Entries: one per page = 8192 ✓.
- Entry size $= 9 + 3 = 12$ bits; table $= 8192 \times 12 = 98\,304$ bits ✓.
- $0x123456$: offset $= 0x456$ (low 11 bits); VPN $= 0x123456 >> 11 = 0x246$ ✓.

3.2

- Walk time per miss: 20 ns; faults add $0.01 \times 4\,\text{ms} = 40\,\mu\text{s}$ per walk ✓.
- AMAT $= 0.98 \times 1 + 0.02 \times (20 + 40\,000) \approx 801$ ns ✓.
- At 0.1%: $0.02 \times (20 + 4000) \rightarrow$ AMAT $= 0.98 + 80.4 \approx 81.4$ ns ✓.
- The fault term dominates both answers; lesson: the hierarchy tolerates rarity, not frequency – locality is the budget ✓.