UNIVERSITY COMPUTER SCIENCE • EXERCISE SHEET

5.3 Hashing

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS load factor • collision

Objective

Predict hash-table costs from the load factor and the resolution strategy.

1 Key knowledge and worked examples

■ **The spread**

$h(k) = k \bmod 13$ sends $4, 17, 30, 43$ all to bucket 4 ✓. With keys $[4, 17, 30, 43, 12, 25, 38]$ the load factor is $\alpha = 7/13 \approx 0.54$ – also the average chain length ✓.

■ **Worked example: probing the cluster**

Linear probing places $4 \to 4$, $17 \to 5$, $30 \to 6$, $43 \to 7$ at a cost of $1 + 2 + 3 + 4 = 10$ probes ✓ – clustering grows the runs ✓.

2 Practice

2.1 Give $h(k)$ for $k = 29, 55, 68$ under mod 13. [3]

2.2 Twelve keys in thirteen buckets: state α and the average chain length. [3]

2.3 State why a prime modulus beats a power of two when keys share low bits. [3]

3 Exam-style questions

3.1 A hash table has $m = 11$ buckets with chaining, and the keys $6, 17, 28, 39, 50$ arrive in order.

Compute each key's bucket, and give every non-empty chain after all insertions. State α, the expected successful-search cost $1 + \alpha/2$, and the exact worst-case comparison count for searching 50 in your table. [8]

3.2 The same five keys enter an open-addressed table of 11 slots with linear probing.

Give the slot each key lands in and the probe count for each insertion, with the running total. State the total had the keys hashed to five distinct buckets. Name the effect that makes the first total larger, and the load-factor policy that controls it. [7]

Solutions

2.1

- $29 \bmod 13 = 3$; $55 \bmod 13 = 3$; $68 \bmod 13 = 3$ ✓.

2.2

- $\alpha = 12/13 \approx 0.92$ ✓.
- The average chain length equals $\alpha \approx 0.92$ ✓.

2.3

- A power-of-two modulus keeps only the low bits – keys sharing them collide wholesale ✓.
- A prime spreads the residue across all bits of the key ✓.

3.1

- Buckets: $6 \bmod 11 = 6$; $17 \bmod 11 = 6$; $28 \bmod 11 = 6$; $39 \bmod 11 = 6$; $50 \bmod 11 = 6$ ✓.
- One chain: $6 \to 17 \to 28 \to 39 \to 50$ in bucket 6 ✓.
- $\alpha = 5/11 \approx 0.45$; expected successful cost $1 + 0.45/2 \approx 1.23$ comparisons ✓.
- Worst case for 50: five comparisons – the whole chain ✓.

3.2

- Slots: $6 \to 6$; 17 probes $6, \to 7$; $28 \to 6, 7, \to 8$; $39 \to 6, 7, 8, \to 9$; $50 \to 6, 7, 8, 9, \to 10$ ✓.
- Probe counts $1, 2, 3, 4, 5$ – running total 15 ✓.
- Five distinct buckets would cost exactly 5 probes total ✓.
- The effect is clustering: the occupied run grows the probe path ✓.
- Policy: keep α under about 0.75 and resize ✓.

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