UNIVERSITY COMPUTER SCIENCE • EXERCISE SHEET

2.6 Discrete probability

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS sample space • Bayes' theorem

Objective

Count outcomes; condition correctly; run Bayes.

1 Key knowledge and worked examples

■ **The rules**

$P(E) = |E|/|\Omega|$ on equally likely outcomes; $P(A \mid B) = P(A \cap B)/P(B)$; independence multiplies ✓.

■ **Worked example: Bayes**

1 % disease, Se = Sp = 0.9: $P(\text{dis} \mid +) = \frac{0.009}{0.108} = 1/12 \approx 8.3\ \%$ ✓ – the tree makes the rescaling visible.

2 Practice

2.1 One fair die: $P(\text{even})$? [3]

2.2 Two coins: $P(\text{two heads} \mid \text{at least one head})$? [3]

2.3 Independent events with $P(A) = 0.5$, $P(B) = 0.4$ – $P(A \cap B)$? [3]

3 Exam-style questions

3.1 Two fair dice, $n = 36$ pairs.

Compute $P(\text{sum} = 5) = 4/36$, $P(\text{sum} = 7) = 6/36$ and $P(\text{sum} = 5 \text{ or } 7) = 10/36$ (disjoint – add) ✓. Then $P(\text{at least one 6})$ by the complement: $1 - (5/6)^2 = 11/36$ ✓. Verify the identity $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ on $A =$ first die 6, $B =$ second die 6: $1/6 + 1/6 - 1/36 = 11/36$ ✓. [8]

3.2 A box: 3 red, 2 blue. Two draws without replacement.

Build the outcome tree with products on the branches; compute $P(\text{two red}) = \frac{3}{5} \cdot \frac{2}{4} = \frac{3}{10}$, $P(\text{one of each}) = \frac{3}{10} + \frac{3}{10} = \frac{3}{5}$ ✓. Then Bayes: given the second draw is red, $P(\text{first red} \mid \text{second red}) = \frac{3/10}{3/10+3/10} = \frac{1}{2}$ ✓ – state why it differs from the unconditional 3/5.

[7]

Solutions

2.1

- $3/6 = 1/2$ ✓.

2.2

- Outcomes with at least one head: HH, HT, TH – $1/3$ ✓.

2.3

- $0.5 \times 0.4 = 0.2$ ✓.

3.1

- Sum 5: (1,4),(2,3),(3,2),(4,1) – $4/36$; sum 7: $6/36$ ✓.
- Disjoint events add: $10/36$ ✓.
- At least one 6 by complement: $1 - 25/36 = 11/36$ ✓.
- Check: $1/6+1/6-1/36 = 6/36+6/36-1/36 = 11/36$ ✓ – inclusion-exclusion agrees.

3.2

- Branches: RR $\frac{3}{5} \cdot \frac{2}{4} = \frac{3}{10}$; RB and BR $\frac{3}{10}$ each; BB $\frac{1}{10}$ ✓.
- One of each: $\frac{3}{10} + \frac{3}{10} = \frac{3}{5}$ ✓.
- Bayes: $P(R_1 \mid R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{3/10}{3/10+3/10} = \frac{1}{2}$ ✓.
- It differs from $3/5$ because a red second draw is evidence – the condition reshapes the sample space ✓.