UNIVERSITY COMPUTER SCIENCE • EXERCISE SHEET

2.5 Graphs and trees

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS handshake lemma • adjacency list

Objective

Represent graphs; apply handshake; walk trees.

1 Key knowledge and worked examples

■ **Representations**

Adjacency matrix: n^2 space, $O(1)$ edge test. Adjacency list: $O(|V| + |E|)$ – sparse-friendly ✓. Tree: connected + acyclic, $|E| = |V| - 1$.

■ **Worked example: handshake**

Every edge contributes 2 to the degree sum: $\sum_v \deg(v) = 2|E|$ – so the number of odd-degree vertices is even ✓. 12-vertex tree, 5 leaves: internal degree sum $22-5 = 17$.

2 Practice

2.1 Edges in a 9-vertex tree? [3]

2.2 A graph has degrees 3, 3, 2, 2 – how many edges? [3]

2.3 Can exactly one vertex have odd degree? Why not? [3]

3 Exam-style questions

3.1 A graph has 7 vertices with degrees 4, 4, 3, 3, 3, 2, 2.

Check the handshake total (21 – odd, so this degree list is IMPOSSIBLE) ✓. Correct the list by changing one entry so the sum is even and equals $2|E|$ with $|E| = 11$ (e.g. change a 2 to a 3, total 22) ✓. State the corollary used (odd-degree count even) and draw-the-check: any proposed drawing must satisfy it ✓. [8]

3.2 A 10-vertex tree has 4 leaves.

Compute $|E| = 9$ and the total degree sum 18 ✓. The 6 internal vertices carry degree sum $18 - 4 = 14$; state the minimum degree of an internal vertex (2, since one neighbour must lead onward and one back) and derive the maximum possible number of degree-2 internals (if x are degree 2 and the rest degree 3: $2x + 3(6 - x) = 14$ gives $x = 4$) ✓. [7]

Solutions

2.1

- $9 - 1 = 8$ ✓.

2.2

- Sum 10, so $|E| = 5$ ✓.

2.3

- No – the degree sum must be even, so odd degrees come in pairs ✓.

3.1

- Total $4 + 4 + 3 + 3 + 3 + 2 + 2 = 21$ – odd, violating handshake ✓.
- Change one 2 to a 3: total $22 = 2 \times 11$ edges ✓.
- Corollary: the count of odd-degree vertices is even – the original list had three odd entries (impossible) ✓.
- Any drawing must satisfy handshake; the arithmetic screens drawings before pencil moves ✓.

3.2

- $|E| = 10 - 1 = 9$; degree sum $= 2 \times 9 = 18$ ✓.
- Internal sum: $18 - 4 = 14$ across 6 vertices ✓.
- Minimum internal degree 2 (path through) ✓.
- Let x degree-2 and $6 - x$ degree-3: $2x + 18 - 3x = 14 \Rightarrow x = 4$ ✓.

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