UNIVERSITY COMPUTER SCIENCE • EXERCISE SHEET

2.4 Counting and combinatorics

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS permutation • combination

Objective

Choose the rule; count two ways; run inclusion-exclusion.

1 Key knowledge and worked examples

■ The rules

Disjoint alternatives ADD; sequential choices MULTIPLY ✓. $P(n,r) = \frac{n!}{(n-r)!}$ ordered; $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$ unordered.

■ Worked example: inclusion-exclusion

Students: 45 tea, 35 coffee, 30 cocoa; pairs 12, 10, 8; triple 5. At least one: $45+35+30-12-10-8+5=85$; none: $100-85=15$ ✓.

2 Practice

2.1 Committee of 3 from 9 – how many? [3]

2.2 Arrange 5 distinct books on a shelf – how many orders? [3]

2.3 $|A| = 12$, $|B| = 9$, $|A \cap B| = 4$ – $|A \cup B|$? [3]

3 Exam-style questions

3.1 A password is 4 characters from 26 letters and 10 digits, with at least one digit required (repetition allowed).

Count without restriction (36^4), subtract all-letter passwords (26^4) – the complement method ✓. Compute both powers and the final answer ($36^4 - 26^4 = 1679616 - 456976 = 1222640$) ✓. Then count the version with NO repetition allowed ($P(36,4) - P(26,4) = 1054920$) ✓. [8]

3.2 Among 60 students: 30 read fiction, 25 non-fiction, 20 poetry; 12 fiction and non-fiction, 9 fiction and poetry, 7 non-fiction and poetry; 4 all three.

Compute at-least-one by inclusion-exclusion ($30+25+20-12-9-7+4=51$) and exactly-two categories ($12-4+9-4+7-4=16$) ✓. Verify the partition: exactly-one $= 51-16-4=31$ and none $= 60-51=9$ – the four numbers sum to 60 ✓. [7]

Solutions

2.1

- $\binom{9}{3} = 84$ ✓.

2.2

- $5! = 120$ ✓.

2.3

- $12 + 9 - 4 = 17$ ✓.

3.1

- Without restriction: each of 4 slots picks from 36 – 36^4 ✓.
- All-letter: 26^4; at least one digit: subtract – complement ✓.
- $36^4 = 1679616$; $26^4 = 456976$; difference 1222640 ✓.
- No repetition: $36 \cdot 35 \cdot 34 \cdot 33 = 1413720$; all-letter no repetition $26 \cdot 25 \cdot 24 \cdot 23 = 358800$; answer 1054920? – recompute: $1413720 - 358800 = 1054920$ ✓.

3.2

- At least one: $30 + 25 + 20 - 12 - 9 - 7 + 4 = 51$ ✓.
- Exactly two: each pair count minus the triple, summed: $(12-4)+(9-4)+(7-4) = 16$ ✓.
- Exactly one: $51 - 16 - 4 = 31$; none: 9 ✓.
- Partition check: $31 + 16 + 4 + 9 = 60$ ✓.

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