UNIVERSITY COMPUTER SCIENCE • EXERCISE SHEET

2.2 Sets, functions and relations

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS power set • injective • equivalence relation

Objective

Compute with sets; classify functions; test relations.

1 Key knowledge and worked examples

■ The toolkit

Union, intersection, difference, complement; $|\mathcal{P}(A)| = 2^{|A|}$ – subsets pair with bit strings ✓. Injective (distinct inputs, distinct outputs), surjective (every output hit), bijective (both); $g \circ f$ applies f first.

■ Worked example: relation testing

On $\{1,2,3\}$: $R = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$ – reflexive (all loops present), symmetric (pairs come in twos), transitive ($(1,2),(2,1)$ demand $(1,1)$ – present) ✓.

2 Practice

2.1 How many subsets does a 4-element set have? [3]

2.2 Is $f(n) = n + 1$ on the integers injective, surjective, both? [3]

2.3 What does a symmetric relation's matrix look like? [3]

3 Exam-style questions

3.1 $A = \{1,2,3,4\}$, $B = \{2,4,6\}$.

Compute $|A \cup B|$, $|A \cap B|$, $|A - B|$, $|B - A|$ ✓ and verify $|A \cup B| = |A| + |B| - |A \cap B|$ numerically ✓. Then count the functions from A to B ($3^4 = 81$) and the injections from B to A ($4 \times 3 \times 2 = 24$), naming the rule used each time ✓. [8]

3.2 Three relations on $\{a,b,c\}$, given as sets of pairs.

$R_1 = \{(a,a),(b,b),(c,c)\}$: reflexive, symmetric, antisymmetric, transitive ✓. $R_2 = \{(a,b),(b,a)\}$: symmetric not reflexive; is it transitive? – no: $(a,b),(b,a)$ demand (a,a), absent ✓. R_3 with $(a,b),(b,c)$ only: transitive requires (a,c) – test what is missing ✓. State which of the three is an equivalence relation (only R_1) and why ✓. [7]

Solutions

2.1

- $2^4 = 16$ ✓.

2.2

- Bijective – shifts never merge inputs and every integer is hit ✓.

2.3

- Symmetric about the diagonal: matrix = its transpose ✓.

3.1

- $A \cup B = \{1, 2, 3, 4, 6\}$: 5; $A \cap B = \{2, 4\}$: 2 ✓.
- $A - B = \{1, 3\}$: 2; $B - A = \{6\}$: 1 ✓.
- Check: $5 = 4 + 3 - 2$ ✓ – inclusion-exclusion holds.
- Functions $A \to B$: each of 4 inputs picks one of 3 outputs – $3^4 = 81$ (multiplication rule) ✓.
- Injections $B \to A$: 4 choices, then 3, then 2 – 24 ✓.

3.2

- R_1: all four properties hold (identity relation) ✓.
- R_2: symmetric, not reflexive (no loops); transitivity fails on the round trip without (a, a) ✓.
- R_3: transitive requires the composite (a, c) – its absence decides ✓.
- Equivalence = reflexive + symmetric + transitive: only R_1 qualifies ✓.