2.1 Logic and proofs

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS contraposition • contradiction

Objective

Translate claims; choose and run the proof shape.

1 Key knowledge and worked examples

■ **The equivalences**

$p \to q \equiv \neg p \lor q$; contraposition $p \to q \equiv \neg q \to \neg p$; De Morgan $\neg(p \land q) \equiv \neg p \lor \neg q$; negation swaps $\forall$ with $\exists$ ✓.

■ **Worked example: strategy named**

If n^2 is even then n is even – CONTRAPOSITION: $n = 2k + 1$ gives $n^2 = 2(2k^2 + 2k) + 1$, odd ✓. Translating a claim and negating it are the two routine steps before any shape is chosen.

2 Practice

2.1 State the contrapositive of $p \to q$. [3]

2.2 Negate $\forall x\, P(x)$ and $\exists x\, P(x)$. [3]

2.3 Apply De Morgan to $\neg(r \lor s)$. [3]

3 Exam-style questions

3.1 Three claims to prove, one per shape.

(i) If $3n + 2$ is odd then n is odd – name the shape, prove it ✓. (ii) $\sqrt{2}$ is irrational – name the shape (contradiction), write the parity case analysis on $p^2 = 2q^2$ ✓. (iii) If $xy = 0$ then $x = 0$ or $y = 0$ – CONTRAPOSITION of the conclusion: show $x \neq 0$ and $y \neq 0$ forces $xy \neq 0$ ✓. For each: state the strategy BEFORE the proof, as the course rule demands. [8]

3.2 An argument is presented: the premises are $p \to q$ and p; the conclusion is q – and a second one with premises $p \to q$ and q, concluding p.

Verify the first with a two-row truth table (modus ponens: rows where both premises hold force q) ✓. Show the second is INVALID – exhibit the row $p = F$, $q = T$ where premises hold and conclusion fails (that shape is affirming the consequent) ✓. [7]

Solutions

2.1

- $\neg q \to \neg p$ ✓.

2.2

- $\exists x\, \neg P(x)$; $\forall x\, \neg P(x)$ ✓.

2.3

- $\neg r \wedge \neg s$ ✓.

3.1

- (i) Contraposition: n even $\Rightarrow n = 2k \Rightarrow 3n + 2 = 6k + 2$ even ✓.
- (ii) Contradiction: $\sqrt{2} = p/q$ lowest terms $\Rightarrow p^2 = 2q^2 \Rightarrow p$ even $\Rightarrow q$ even $\Rightarrow$ terms not lowest – absurd ✓.
- (iii) Contrapositive of the conclusion: $x \neq 0$ and $y \neq 0 \Rightarrow xy \neq 0$ – negates the premise directly ✓.
- Strategy named before each proof, per the course rule ✓.

3.2

- Modus ponens table: only row $p = T, q = T$ satisfies both premises; $q = T$ there ✓.
- Second: row $p = F, q = T$ satisfies $p \to q$ and q but p fails – invalid ✓.
- The invalid shape is affirming the consequent; a truth-table row is the proof of invalidity ✓.

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