13.1 Search

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS search tree • heuristic

Objective

Run A* by hand and read admissibility as a contract.

1 Key knowledge and worked examples

■ **The machinery**

$f = g + h$; BFS shortest when steps equal; DFS frugal ✓. Course graph: D→A (4), D→B (2), A→C (3), B→C (1), C→G (5), B→G (9), with $h = \{A{:}8, B{:}6, C{:}5, G{:}0\}$ ✓.

■ **Worked example: the trace**

A* pops D (0), B (8), C (8), G (8): optimal 8 via D→B→C→G – four expansions ✓.

2 Practice

2.1 What does admissible mean, in symbols? [3]

2.2 What is A*'s f at the moment it pops the goal? [3]

2.3 Why is A* with an admissible h optimal at the FIRST goal pop? [3]

3 Exam-style questions

3.1 Course graph above. Animate A* from D.

Give every pop with its (g, h, f) at pop time. Give the expansions count and the optimal path with its cost. State which edge A* never uses and why it still cannot be missed. Contrast the BFS visit order. [8]

3.2 Replace h with $h'(C) = 12$ (everything else equal).

Re-derive the pop order and the returned cost. State why optimality fails, naming the violated property. Give the smallest change to the graph that would restore optimality despite h' (the direct B→G edge). State the general lesson about trusting estimates. [7]

Solutions

2.1

- $h(n) \leq h^*(n)$ for every node: never overestimates the true remaining cost ✓.

2.2

- $f(goal) = g(goal)$, since $h(goal) = 0$ ✓.

2.3

- That f is a lower bound for every open node: no unexpanded path could be cheaper ✓.

3.1

- Pops: D ($g0, h0, f0$); B ($g2, h6, f8$); C ($g3, h5, f8$); G ($g8, h0, f8$) ✓.
- Four expansions; path D→B→C→G at cost $2 + 1 + 5 = 8$ ✓.
- A (f = 12) is never popped: its subtree cannot beat the goal's bound – and the unused A→C edge is exactly what that bound excludes ✓.
- BFS order: D, A, B, C, G (level by level) – five visits, no cost awareness ✓.

3.2

- With $h'(C) = 12$: C carries $f = 3 + 12 = 15$; pops become D, B, then G (through B, $f = 11$) – returned cost 11 ✓.
- Admissibility is violated at C ($12 > 5$): the goal pops while a cheaper open path exists ✓.
- Restoring fix: remove the direct B→G edge – then G is reachable only through C, and even the lying h' must pop C first ✓.
- Lesson: an estimate is a contract; optimality is only as honest as the tightest overestimate in the graph ✓.

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