UNIVERSITY COMPUTER SCIENCE • EXERCISE SHEET

11.1 Automata and regular languages

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS deterministic finite automaton • pumping lemma

Objective

Trace DFAs exactly and use the pumping lemma as a ceiling.

1 Key knowledge and worked examples

■ The machine

States plus a transition table; acceptance only at end of input ✓. The ends-in-ab DFA: `aabab` accepts, `aba` rejects ✓.

■ Worked example: the ceiling

Pumping: any long accepted string has a pumpable y; $a^n b^n$ breaks when pumped – not regular ✓.

2 Practice

2.1 In the ends-in-ab DFA, which state accepts? [3]

2.2 Why is `aba` rejected? [3]

2.3 One line: what does the pumping lemma forbid a regular language from doing? [3]

3 Exam-style questions

3.1 The ends-in-ab DFA over the alphabet a, b has these transitions: q0 on a goes to q1; q1 on a stays q1; q1 on b goes to q2; q2 on a goes to q1; q0 on b stays q0; q2 on b goes to q0. The accepting state is q2.

Give the full state trace of `babbab` and the verdict. Give the trace of `babba`. State which two positions of the input could be swapped to flip the verdict, and why. Draw (or describe) the missing 8th transition a complete table needs if any input letter were added. [8]

3.2 Use the pumping lemma to argue $L = \{a^n b^n\}$ is not regular with p as the pumping length.

Choose the string and state its constraint on n. Split it xyz and state what y must contain. Pump once and derive the contradiction. State what this ceiling means for a lexer that must match nested brackets. [7]

exercises.lol exercises.com.cn

Solutions

2.1

- q_2 ✓.

2.2

- It ends in state q_1 – the final `a` leaves the pattern incomplete ✓.

2.3

- It forbids unbounded counting: every long string has a pumpable loop ✓.

3.1

- `babbab`: q0→q0 (b), q0→q1 (a), q1→q2 (b), q2→q0 (b), q0→q1 (a), q1→q2 (b) – accepted ✓.
- `babba`: q0, q0, q1, q2, q1, q1 – rejected ✓.
- Swapping the last two letters flips it: `babbab` keeps ...ab; `babba...` variants end ba or aa – only strings ending ab accept, so the final pair decides ✓.
- A complete table over $\{a, b, c\}$ needs a dead state q_d: q_d loops on every letter, entered on c from anywhere ✓.

3.2

- Choose $a^p b^p$ ($n = p$ forced by the lemma's threshold) ✓.
- $|xy| \leq p$ keeps y inside the a-block: $y = a^k$, $k \geq 1$ ✓.
- Pump once: $xy^2z = a^{p+k}b^p$ – more a's than b's, not in L ✓.
- Ceiling for lexers: nested brackets need counting, so no regex (DFA) lexer can match them – the parser above must take over ✓.

exercises.lol exercises.com.cn