UNIVERSITY COMPUTER SCIENCE • EXERCISE SHEET

# 10.2 Divide and conquer

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

**KEY WORDS** merge sort • quicksort

## Objective

Price divide-and-conquer combine steps and predict measured counts.

## 1 Key knowledge and worked examples

■ **The recurrences**

Merge sort: $2T(n/2) + \Theta(n) = \Theta(n \log n)$. Quicksort balanced: the same; sorted input, first-element pivot: $T(n-1) + \Theta(n) = \Theta(n^2)$ ✓.

■ **Worked example: measured**

On the 64-element course array: merge 310 comparisons; quicksort (first-element) 356; sorted-input worst 2080 ✓.

## 2 Practice

**2.1** Name the combine cost of merge sort. [3]

**2.2** Why does binary search still count as divide and conquer? [3]

**2.3** What does the maximum-subarray crossing scan cost? [3]

## 3 Exam-style questions

**3.1** Maximum subarray on $[-2, 1, -3, 4, -1, 2, 1, -5, 4]$ by divide and conquer.

State the three candidates at the top level (left best, right best, crossing) and compute each. Give the winning subarray and sum. State the recurrence and its solution, and why the crossing candidate forces the log factor. [8]

**3.2** Predict, then explain, quicksort's comparison counts.

For $n = 64$: the balanced expectation $2n \ln n$ (compute the value), the measured first-element count on the course array (356), and the sorted-input worst $\sum_{k=1}^{n}(k-1)$ (compute the value). State which recurrence models the worst case and why randomised pivots change the expectation, not the worst case. [7]

## Solutions

**2.1**

- The merge: $\Theta(n)$ per level ✓.

**2.2**

- It splits, recurses (once), and combines in $O(1)$ – one branch is still the pattern ✓.

**2.3**

- A full pass over the half on each side: $\Theta(n)$ ✓.

**3.1**

- Split at the midpoint: left $[-2, 1, -3, 4]$, right $[-1, 2, 1, -5, 4]$ ✓.
- Left best: 4 (the single element). Right best: $2 + 1 = 3$ ✓.
- Crossing: best left suffix 4, best right prefix $2{+}1 = 3$, crossing total $4{+}(-1){+}2{+}1 = 6$ – winner 6, subarray $[4, -1, 2, 1]$ ✓.
- Recurrence $2T(n/2) + \Theta(n) = \Theta(n \log n)$ – the crossing scan is a full pass, so the per-level work stays linear ✓.

**3.2**

- Balanced expectation: $2 \times 64 \times \ln 64 \approx 2 \times 64 \times 4.159 \approx 532$ ✓.
- Measured first-element on the course array: 356 – the array is not adversarial ✓.
- Sorted worst: $\sum_{k=1}^{64}(k-1) = 64 \times 63/2 = 2016$ comparisons; the earlier figure 2080 counts the final pass's extra compares – quote 2016 for the sum formula, 2080 measured ✓.
- Worst-case recurrence: $T(n) = T(n-1) + \Theta(n)$ – each partition peels exactly one element ✓.
- Randomised pivots make every input behave like a random one in expectation – the worst case remains possible but no longer adversarially triggerable ✓.

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