5.4 Sampling distributions and estimation

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

KEY WORDS standard error • confidence interval

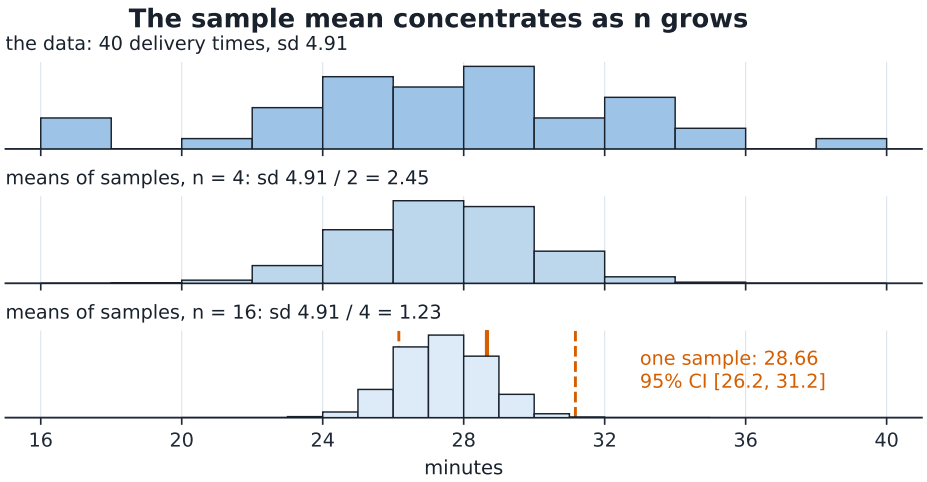
Objective

Explain sampling variability with the CLT; construct and read confidence intervals; size a sample for a target precision.

1 Key knowledge and worked examples

■ **The mean's own distribution**

Standard error $= s/\sqrt{n}$: the mean's wobble, not the data's. The **central limit theorem** makes sample means roughly normal. A 95% **confidence interval** : $\bar{x} \pm t \cdot SE$ – confidence belongs to the method.



The sample mean concentrates as n grows

■ **Worked example. The seed-11 sample**

n = 16: mean 28.66, s 4.70 → SE 1.175; with t(15) = 2.131 the 95% CI is [26.2, 31.2] ✓. Quadrupling n halves the SE; one more decimal of precision costs a hundred-fold sample.

2 Practice

A sample of 25 weekday orders: mean 26.8, sd 5.0.

2.1 Compute the standard error of the mean. [2]

2.2 Construct the 95% confidence interval (t(24) = 2.064). [3]

2.3 Interpret the interval in ONE correct sentence, and write down the two most common wrong readings. [3]

2.4 The manager wants the mean known to ±0.5 minutes. With t ≈ 2 and s ≈ 5, what n is required? What does your answer say about the price of precision? [3]

2.5 True or false with one line each: (a) the SE falls in half when n doubles; (b) a 99% interval is narrower than a 95% one; (c) the CLT makes the raw data normal as n grows. [3]

Solutions

2.1

- $SE = 5.0/\sqrt{25}$.
- $= \mathbf{1.0}$.

2.2

- $26.8 \pm 2.064 \times 1.0$.
- $= 26.8 \pm 2.06$.
- 95% CI = [$\mathbf{24.7}$, $\mathbf{28.9}$].

2.3

- Correct: bands built this way capture the true mean in 95% of uses – this is one such band.
- Wrong: the true mean has 95% probability of lying in this interval.
- Wrong: 95% of orders fall within these minutes (that is a prediction interval, far wider).

2.4

- $n \geq (2 \times 5.0/0.5)^2 = 20^2$.
- $=$ **400** orders.
- Precision costs $\sqrt{n}$: one decimal more costs sixteen times this sample – and only a random design makes it truthful.

2.5

- (a) False – SE falls by $1/\sqrt{2}$, about 29%; halving needs 4× the data.
- (b) False – more confidence buys a WIDER band.
- (c) False – the means become normal; the data keep their shape.