5.3 Probability and distributions

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS binomial distribution • normal distribution

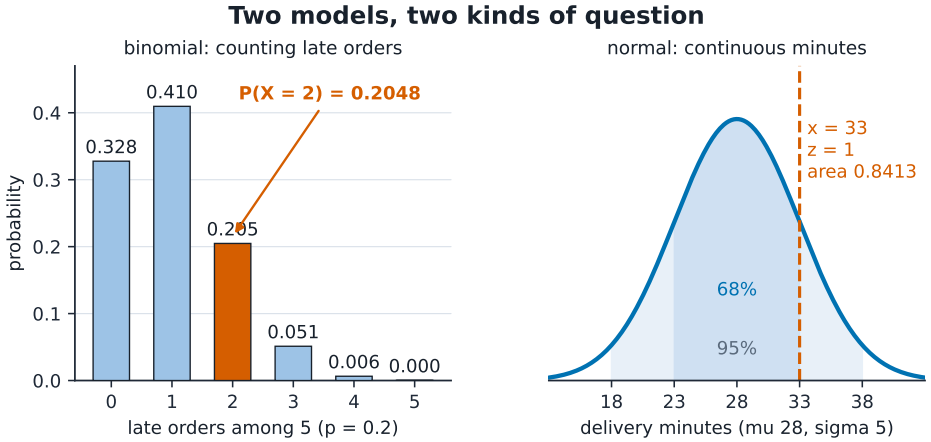
Objective

Apply probability rules; compute binomial and normal probabilities with the model justified; standardise and read tables correctly.

1 Key knowledge and worked examples

■ **Two models, matched to questions**

Counts of successes → **binomial** : $\binom{n}{k}p^k(1-p)^{n-k}$. Symmetric continuous measurements → **normal** : standardise with $z = (x-\mu)/\sigma$ and read Φ. Justify the model before computing.



Counts on the left; continuous minutes on the right

■ **Worked example. The 30-minute promise**

$\mu = 28$, $\sigma = 5$: $P(X > 30) = 1 - \Phi(0.4) = 0.345$ – a third of orders claim the free dessert ✓. Speeding the kitchen to $\mu = 26$ still leaves 0.212. The promise's cost is a probability.

2 Practice

2.1 P(rain) = 0.3, P(rush hour) = 0.4, both together 0.12. Compute P(rain or rush), and state whether the two are independent. [3]

2.2 An order is late with probability 0.15. For the next 6 orders, compute P(exactly 1 late) and P(none late). [4]

2.3 Delivery minutes follow N(26, 4). Compute P(an order exceeds 32) and P(it finishes within ±6 minutes of the mean). [4]

2.4 In two lines: why is the binomial the wrong model for "P(this specific order is late)" and what IS the right framing? [2]

2.5 Match each question to its model with one reason each: (a) at least 3 of 25 orders complain; (b) a single order's total weight in grams; (c) which of 4 riders gets the north route. [3]

Solutions

2.1

- $P(\cup) = 0.3 + 0.4 - 0.12 = 0.58$.
- Independence check: $P(\text{rain}) \times P(\text{rush}) = 0.12$.
- Equal to the joint $\rightarrow$ independent.

2.2

- $P(1) = \binom{6}{1}(0.15)(0.85^5) = 6 \times 0.15 \times 0.4437$.
- $= \mathbf{0.399}$.
- $P(0) = 0.85^6$.
- $= \mathbf{0.377}$.

2.3

- $z = (32 - 26)/4 = 1.5$; tail $= 1 - 0.9332 = \mathbf{0.0668}$.
- Within ± 6: $z = \pm 1.5 \rightarrow 0.9332 - 0.0668 = \mathbf{0.8664}$.

2.4

- The binomial counts successes among MANY trials.
- A single order is one Bernoulli trial: "late" is an event with probability 0.15 – the binomial appears only when counting across orders.

2.5

- (a) Binomial(25, complaint rate) – counting successes.
- (b) Normal – continuous, roughly symmetric measurement (weights pile around a mean).
- (c) Equally likely outcomes – direct counting, 1 in 4.

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