Circle equations, arcs and unit-circle trigonometry

Original skill preparation; versioned source scope

Circle Equations, Arcs and Unit-Circle Trigonometry

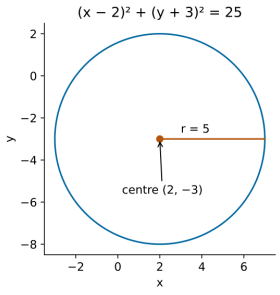
- Read and write a **circle equation** 圆方程, completing the square when needed.
- Convert between degrees and the **radian** 弧度.
- Find **arc length** 弧长 and the area of a **sector** 扇形.
- Use the **tangent** 切线 and inscribed-angle facts, and the unit circle.

The equation of a circle

- The centre is (h, k) and the radius r. Watch the signs: $(y+3)^2$ means $k=-3$.
- From $x^2+y^2+Dx+Ey+F=0$, complete the square in x and in y.
- A point is on the circle if it satisfies the equation, inside if the left side is less than r^2.

$$(x-h)^2+(y-k)^2=r^2$$

Figure: The equation of a circle



- The circle $(x - 2)^2 + (y + 3)^2 = 25$, with centre $(2, -3)$ and radius 5
- Match each part of the diagram to the text.

Radians, arcs and sectors

- $180^\circ = \pi$ radians, so multiply degrees by $\dfrac{\pi}{180}$.
- With θ in radians: **arc length** $s = r\theta$, **sector** area $A = \frac{1}{2}r^2\theta$.
- In degrees: arc $= \dfrac{\theta}{360} \times 2\pi r$, sector $= \dfrac{\theta}{360} \times \pi r^2$.

Angles and tangents

- An inscribed angle is half the central angle on the same arc. An angle in a semicircle is 90°.
- A tangent touches the circle once and is perpendicular to the radius at that point.
- So the tangent's slope is the negative reciprocal of the radius's slope.

The unit circle

- On a circle of radius 1 centred at the origin, the point at angle θ is $(\cos\theta, \sin\theta)$.
- So $\cos^2\theta + \sin^2\theta = 1$.
- Signs: both positive in quadrant I; $\cos$ negative in II and III; $\sin$ negative in III and IV.

Worked example: completing the square

- Group: $(x^2 - 8x) + (y^2 + 2y) = -8$.
- Complete: $(x-4)^2 - 16 + (y+1)^2 - 1 = -8$, so $(x-4)^2 + (y+1)^2 = 9$.
- Centre $(4, -1)$, radius 3. Check $(7, -1)$: $3^2 + 0^2 = 9$, so it is on the circle.

Worked example: a sector

- Radians: $135 \times \dfrac{\pi}{180} = \dfrac{3\pi}{4}$.
- Arc: $8 \times \dfrac{3\pi}{4} = 6\pi$ cm. Area: $\frac{1}{2} \times 64 \times \dfrac{3\pi}{4} = 24\pi$ cm^2.
- An inscribed angle on the same arc is $135 \div 2 = 67.5°$.

Common errors

- Getting the sign of the centre wrong.
- Giving r^2 as the radius.
- Using degrees in $s = r\theta$.

Common errors (2)

- Forgetting the 90° between a tangent and a radius.
- Choosing the wrong sign for $\cos$ or $\sin$ in a quadrant.

Key terms

- **circle equation**: $(x-h)^2+(y-k)^2=r^2$, for centre (h,k) and radius r.
- **radian**: The angle at the centre made by an arc equal in length to the radius.
- **arc length**: The distance along part of a circle.
- **sector**: The region between two radii and an arc.

Check yourself

- Convert $60°$ to radians.
- Find the **arc length** for radius 6 and angle $\frac{\pi}{3}$.
- A circle has centre $(0, 0)$ and passes through $(3, 4)$. What is the slope of the tangent there?