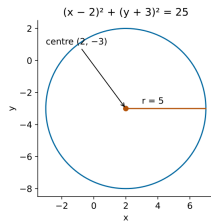


# Circle equations, arcs and unit-circle trigonometry

Original skill preparation; versioned source scope

## Method and conditions



- Complete both squares in a circle equation .
- Complete the square; radius; radians; right-triangle ratios.

## Worked method 1

### Read and reason

Complete both squares in a circle equation .  $x^2 + y^2 + 6x - 8y = 0$  becomes  $(x + 3)^2 + (y - 4)^2 = 25$ . The centre is  $(-3, 4)$  and radius is 5. Arc length is  $s = r\theta$  only for radian angle  $\theta$ .

## Original context 1

Read and reason

## Task context

### Read and reason

Each task below states its own complete data. Values may differ from the worked example.

## Attempt 1.1

## Read and reason

Find the centre and radius of  $x^2 + y^2 - 8x + 2y + 8 = 0$ . Does  $(7, -1)$  lie on it?

## Reasoning 1.1

### Read and reason

Complete squares to get  $(x - 4)^2 + (y + 1)^2 = 9$ . Centre  $(4, -1)$ , radius 3.  
Substitution of  $(7, -1)$  gives  $3^2 + 0^2 = 9$ , so it is on the circle.

## Attempt 2.1

### Read and reason

A sector has radius 8 cm and angle  $135^\circ$ . Find its angle in radians, arc length and area.  
An inscribed angle elsewhere on the circle intercepts this same arc; find it.

## Reasoning 2.1

### Read and reason

$\theta = 135\pi/180 = 3\pi/4$  radians.  $s = r\theta = (8 \text{ cm})(3\pi/4) = 6\pi \text{ cm}$ .

$A = r^2\theta/2 = (8 \text{ cm})^2(3\pi/4)/2 = 24\pi \text{ cm}^2$ . The inscribed angle is half the central angle: 67.5 degrees.

## Attempt 3.1

## Read and reason

A unit-circle point is in quadrant III and has  $y = -1/2$ . Find its  $x$ -coordinate and one angle between  $0^\circ$  and  $360^\circ$ .

## Reasoning 3.1

### Read and reason

$x^2 + y^2 = 1$  gives  $x^2 = 3/4$ . Quadrant III requires  $x < 0$ , so  $x = -\sqrt{3}/2$ . The point is at 210 degrees, with reference angle 30 degrees.

## Transfer and limits

Read and reason

Complete the square; radius; radians; right-triangle ratios.

## Check yourself

### Read and reason

Close the worked reasoning. Explain why the method fits the final transfer task, reconstruct its conclusion, and state a condition that would change your answer. Then check the full reasoning in the reference.