Scale factors, angle relationships and triangles

Original skill preparation; versioned source scope

Scale Factors, Angle Relationships and Triangles

- Use angle facts for lines, parallel lines and triangles.
- Use **similarity** 相似 and the **scale factor** 比例因子 for lengths, areas and volumes.
- Recognise **congruence** 全等 conditions.
- Use Pythagoras's theorem, special right triangles and sine, cosine and tangent, and find areas with the **perpendicular height** 垂直高度.

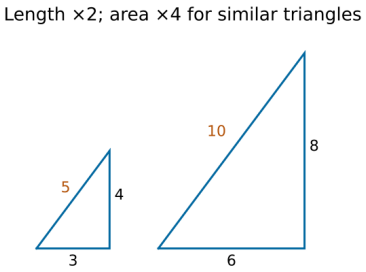
Angle facts

- Angles on a straight line add to 180°; angles around a point add to 360°; vertical angles are equal.
- With parallel lines cut by a transversal, corresponding angles and alternate angles are equal; co-interior angles add to 180°.
- The angles of a triangle add to 180°; an exterior angle equals the sum of the two opposite interior angles.

Similar and congruent triangles

- Triangles are similar if their angles are equal (two pairs are enough). Corresponding sides are in the same ratio k.
- Lengths scale by k, areas by k^2, volumes by k^3.
- Triangles are congruent (same shape and size) by SSS, SAS, ASA, AAS or, for right triangles, hypotenuse–leg. Angles alone fix shape, not size.

Figure: Similar and congruent triangles



- Two similar right triangles, with sides 3, 4, 5 and 6, 8, 10
- Match each part of the diagram to the text.

Right triangles

- 45°–45°–90° sides are in the ratio $1 : 1 : \sqrt{2}$; 30°–60°–90° sides are $1 : \sqrt{3} : 2$.
- $\sin x^\circ = \cos(90 - x)^\circ$, since the two acute angles of a right triangle add to 90°.
- Common whole-number triangles: 3–4–5, 5–12–13 and their multiples.

$$a^2 + b^2 = c^2 \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Area

- Triangle area $= \frac{1}{2} \times$ base $\times$ **perpendicular height**, where the height is at right angles to the base, not a slanted side.
- Composite shapes: split into rectangles and triangles, or subtract a cut-out.

Worked example: similar containers

- Area ratio $= \dfrac{216}{96} = 2.25 = k^2$, so $k = 1.5$.
- Volume ratio $= k^3 = 3.375$.
- Larger volume $= 160 \times 3.375 = 540$ cm^3. Using the area ratio would wrongly give 360 cm^3.

Worked example: a right triangle

- Other leg: $\sqrt{13^2 - 5^2} = \sqrt{144} = 12$ cm.
- Area $= \frac{1}{2} \times 5 \times 12 = 30$ cm^2.
- For the angle θ opposite the 5 cm leg, $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$.

Common errors

- Using the length ratio for areas or volumes.
- Matching sides of similar triangles in the wrong order.
- Using a slanted side as the height.

Common errors (2)

- Assuming lines are parallel because they look parallel.
- Assuming equal angles make triangles congruent.

Key terms

- **similarity**: Having the same shape, with all lengths in the same ratio.
- **scale factor**: The number every length is multiplied by.
- **congruence**: Having the same shape and size.
- **perpendicular height**: The height measured at right angles to the base.

Check yourself

- Two angles of a triangle are $50°$ and $60°$. Find the third, and the exterior angle next to it.
- Lengths are multiplied by 3. What happens to area?
- $\sin 30° = \cos ?$