Probability from a table and a restricted group

Original skill preparation; versioned source scope

Probability from a Table and a Restricted Group

- Find probabilities from a **two-way table** 双向表.
- Find a **conditional probability** 条件概率, the probability within a restricted group.
- Decide whether two events are **independent events** 独立事件.
- Use "and", "or" and "not" correctly.

Probability from counts

- Probabilities lie between 0 and 1. $P(\text{not } A) = 1 - P(A)$.
- In a table, the total is usually the grand total, unless the question restricts the group.

$$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}}$$

Restricted groups

- "Given that", "of those who", "among" all restrict the group: the denominator becomes that row or column total.
- $P(A \mid B) = \dfrac{\text{number in both } A \text{ and } B}{\text{number in } B}$.
- $P(A \mid B)$ and $P(B \mid A)$ are usually different, because they have different denominators.

Figure: Restricted groups

Original counts: condition on the requested group

Group	Lunch	No lunch	Total
Bus	18	12	30
Not bus	12	8	20
Total	30	20	50

- **Two-way table**: bus users 18 with lunch and 12 without (30); non-bus users 12 with lunch and 8 without (20); total 50
- Match each part of the diagram to the text.

Independence

- Events A and B are independent if knowing B does not change the probability of A: $P(A \mid B) = P(A)$.
- Independent is not the same as disjoint. Disjoint events cannot happen together; if both have positive probability, they are not independent.

"Or" and "and"

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, so the overlap is not counted twice.
- In a table, it is often easier to count the cells directly.
- Expected count = probability $\times$ number of trials.

Worked example: the lunch table

- $P(\text{lunch}) = \dfrac{18+12}{50} = \dfrac{30}{50} = 0.6.$
- $P(\text{lunch} \mid \text{bus}) = \dfrac{18}{30} = 0.6.$
- These are equal, so in this table bringing lunch is independent of using the bus.
- $P(\text{bus} \mid \text{lunch}) = \dfrac{18}{30} = 0.6$, and $P(\text{bus}) = \dfrac{30}{50} = 0.6$ as well.

Worked example: a table that is not independent

- $P(\text{tea}) = \dfrac{21}{50}$; $P(\text{tea} \mid \text{cyclist}) = \dfrac{12}{20} = \dfrac{3}{5}$. These differ, so the events are not independent.
- $P(\text{cyclist} \mid \text{tea}) = \dfrac{12}{21} = \dfrac{4}{7}$: a different denominator, so a different answer.
- $P(\text{walker or tea}) = \dfrac{30 + 21 - 9}{50} = \dfrac{42}{50}$; the 9 tea-drinking walkers are counted once.

Common errors

- Using the grand total when the question restricts the group.
- Mixing up $P(A \mid B)$ and $P(B \mid A)$.
- Counting the overlap twice in an "or" question.

Common errors (2)

- Confusing independent with disjoint.
- Reading the wrong row or column of the table.

Key terms

- **two-way table**: A table that counts outcomes by two categories at once.
- **conditional probability**: The probability of an event within a restricted group.
- **independent events**: Events where one happening does not change the probability of the other.

Check yourself

- Of 16 boys, 10 play chess. What is $P(\text{chess} \mid \text{boy})$?
- $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \text{ and } B) = 0.1$. Find $P(A \text{ or } B)$.
- If $P(A \mid B) = P(A)$, what can you say about A and B?