Ratios, unit conversion and successive percentages

Original skill preparation; versioned source scope

Ratios, Unit Conversion and Successive Percentages

- Share amounts in a **ratio** 比.
- Find and compare a **unit rate** 单位率.
- Convert units with conversion factors, including area and volume.
- Work with a **percentage change** 百分比变化, including several changes in a row.

Ratios

- In a ratio $a : b$, the whole has $a + b$ parts. One part is the total divided by $a + b$.
- Ratios can be scaled: $4 : 6 = 2 : 3$.
- A part-to-part ratio is not a fraction of the whole: in $2 : 3$, the first is $\frac{2}{5}$ of the total, not $\frac{2}{3}$.

Rates and unit rates

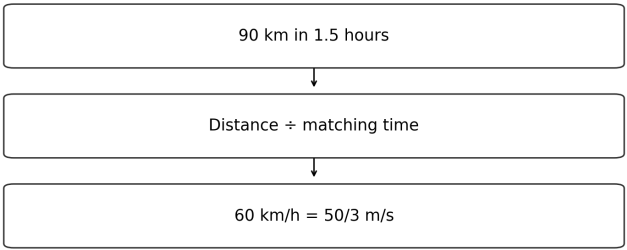
- A rate compares quantities with different units: kilometres per hour, yuan per kilogram.
- A **unit rate** gives the amount for one unit. Divide to find it, then compare like with like.
- Time $=$ amount $\div$ rate.

Unit conversion

- Write each factor as a fraction equal to 1, arranged so the unwanted units cancel.
- For area, square the factor: $1\ \text{m}^2 = 10\,000\ \text{cm}^2$. For volume, cube it.
- Or convert lengths first, then square or cube.

$$60\ \frac{\text{km}}{\text{h}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{h}}{3600\ \text{s}} = \frac{50}{3}\ \frac{\text{m}}{\text{s}}$$

Figure: Unit conversion



- Worked steps: 90 km in 1.5 hours $\rightarrow$ Distance $\div$ matching time $\rightarrow$ 60 km/h $=$ 50/3 m/s
- Match each part of the diagram to the text.

Percentages

- **Percentage change** $= \dfrac{\text{new} - \text{original}}{\text{original}} \times 100\%$.
- An increase of $r\%$ multiplies by $1 + r/100$; a decrease multiplies by $1 - r/100$.
- Successive changes multiply: they do not add.
- Reverse a change by dividing: if 96 is after a 20% rise, the original is $96 \div 1.2 = 80$.

Worked example: a ratio with costs

- Parts: $4 + 7 = 11$, so one part is $33 \div 11 = 3$ L.
- Red $= 12$ L and blue $= 21$ L.
- Cost $= 12 \times 18 + 21 \times 12 = 216 + 252 = 468$ yuan.

Worked example: two percentage changes

- Final price $= 400 \times 0.85 \times 1.10 = 374$ yuan.
- Overall change $= \dfrac{374 - 400}{400} = -0.065$: a 6.5% decrease, not 5%.
- To return to 400 yuan from 374 would need a rise of $26 \div 374 \approx 7.0\%$.

Common errors

- Adding successive percentages instead of multiplying factors.
- Dividing a reverse percentage by the wrong base.
- Using 100 instead of 10,000 for square metres to square centimetres.

Common errors (2)

- Treating a part-to-part ratio as a fraction of the whole.
- Dropping units halfway through a calculation.

Key terms

- **ratio**: A comparison of two quantities by division, written $a : b$.
- **unit rate**: A rate given for one unit of the second quantity.
- **percentage change**: The change divided by the original value, as a percentage.

Check yourself

- Convert 72 km/h to m/s.
- A price falls 30% and then rises 30%. What is the overall change?
- How many cm^2 are in 2.5 m^2?