Nonlinear functions, transformations and growth

Original skill preparation; versioned source scope

Nonlinear Functions: Transformations and Growth

- Read a quadratic in standard, factored and **vertex form** 顶点式, and find the vertex of a **parabola** 抛物线.
- Describe a **transformation** 变换 of a graph from its equation.
- Write and interpret **exponential growth** 指数增长 and decay models.
- Tell linear from exponential change in a table.

Three forms of a quadratic

- Standard, $y = ax^2 + bx + c$: the y-intercept is c.
- Factored, $y = a(x - p)(x - q)$: the x-intercepts are p and q.
- Vertex, $y = a(x - h)^2 + k$: the vertex is (h, k). If $a < 0$, k is the maximum; if $a > 0$, the minimum.
- The vertex lies halfway between the x-intercepts.

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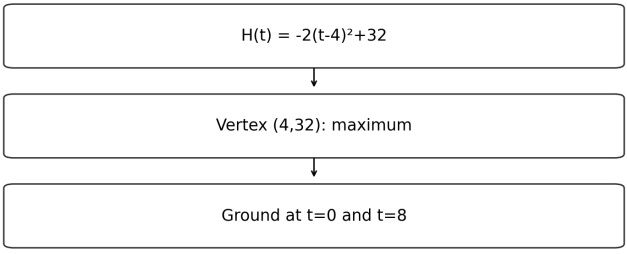
Figure: Three forms of a quadratic



- Worked steps: H(t) = -2(t-4)^2+32 → Vertex (4,32): maximum → Ground at t=0 and t=8
- Match each part of the diagram to the text.

Transformations

- $f(x) + k$ moves the graph up k; $f(x - h)$ moves it right h (note the minus sign).
- $af(x)$ stretches it vertically by a; a negative a also reflects it in the x-axis.
- $f(-x)$ reflects it in the y-axis.
- Track one point, such as the vertex, through each step.

Exponential models

- N_0 is the starting amount; b is the factor for each period of length T.
- Growth of r per cent per period means $b = 1 + r/100$; decay means $b = 1 - r/100$.
- A factor of 1.5 is 50% growth, not 150%.
- Over two periods the factor is b^2, not $2b$.

$$N(t) = N_0\, b^{t/T}$$

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Linear or exponential?

- Equal steps in x give equal differences in y: linear.
- Equal steps in x give equal ratios in y: exponential.
- **Exponential growth** always overtakes linear growth eventually.

Worked example: a projectile

- Ground: $H = 0$ gives $(t-2)^2 = 4$, so $t = 0$ or $t = 4$. Domain $0 \le t \le 4$.
- $a = -3 < 0$, so the vertex $(2, 12)$ is a maximum: 12 m at 2 s.
- At least 9 m: $(t-2)^2 \le 1$, so $1 \le t \le 3$ seconds.

Worked example: growth with a period

- $N(t) = 120(1.5)^{t/2}$, with t in hours.
- $N(4) = 120(1.5)^2 = 120 \times 2.25 = 270$ cells.
- The growth is 50% per two hours. The hourly factor is $\sqrt{1.5} \approx 1.22$, about 22% per hour, not 25%.

Common errors

- Reading the vertex of $y = (x+3)^2$ as $(3, 0)$ instead of $(-3, 0)$.
- Writing a 20% decay factor as 0.2 instead of 0.8.
- Using t instead of $t/2$ when the factor is per two hours.

Common errors (2)

- Calling a table linear because its values increase.
- Forgetting the domain of a real-world model.

Key terms

- **vertex form**: $y = a(x - h)^2 + k$, which shows the vertex (h, k).
- **parabola**: The U-shaped graph of a quadratic function.
- **transformation**: A change to a graph, such as a shift, stretch or reflection.
- **exponential growth**: Growth by the same factor in each equal period.

Check yourself

- Find the vertex of $y = 2(x+3)^2 - 7$. Is it a maximum or a minimum?
- A car's value falls 20% a year. What is the yearly factor?
- The table gives 5, 10, 20, 40 for $x = 0, 1, 2, 3$. Linear or exponential?