Nonlinear equations and mixed systems

Original skill preparation; versioned source scope

Nonlinear Equations and Mixed Systems

- Solve quadratic equations by factoring, completing the square or the formula.
- Use the **discriminant** 判别式 to count real solutions.
- Check each **candidate root** 候选根 from a radical, rational or absolute-value equation.
- Find the **intersection** 交点 points of a line and a **parabola** 抛物线.

Solving quadratics

- Factor when the numbers are small; use the formula otherwise.
- Put everything on one side first. Do not divide by x: in $x^2 = 9x$ that loses the solution $x = 0$.
- Completing the square, $x^2 + 6x = (x+3)^2 - 9$, also gives the vertex.

$$ax^2 + bx + c = 0 \;\Rightarrow\; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant

- $b^2 - 4ac > 0$: two real solutions; the graph crosses the x-axis twice.
- $b^2 - 4ac = 0$: one repeated solution; the graph touches the axis.
- $b^2 - 4ac < 0$: no real solutions.

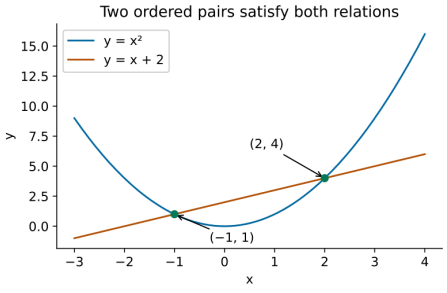
Radical, rational and absolute-value equations

- Radical: isolate the root, square both sides, solve, then check every candidate in the original. Squaring can create a false solution.
- Rational: list the excluded values first, then multiply through by the denominator; reject any candidate that is excluded.
- Absolute value: $|A| = k$ (with $k \geq 0$) splits into $A = k$ or $A = -k$.

Mixed systems

- To solve a line with a parabola, set the two expressions for y equal, giving a quadratic.
- Its discriminant tells you whether the line crosses, touches or misses the parabola.
- Find each y from the simpler (linear) equation.

Figure: Mixed systems



- The parabola $y = x^2$ and the line $y = x + 2$ cross at $(-1, 1)$ and $(2, 4)$
- Match each part of the diagram to the text.

Worked example: a radical equation

- A square root is never negative, so $x \geq 0$.
- Square: $2x + 3 = x^2$, so $x^2 - 2x - 3 = (x - 3)(x + 1) = 0$.
- Candidates: $x = 3$ and $x = -1$.
- Check: $\sqrt{9} = 3$ works; $\sqrt{1} = 1 \neq -1$ fails. Only $x = 3$.

Worked example: a line touching a parabola

- Set equal: $x^2 = x + c$, so $x^2 - x - c = 0$.
- One point means discriminant zero: $(-1)^2 - 4(1)(-c) = 1 + 4c = 0$.
- So $c = -\frac{1}{4}$, and the touching point has $x = \frac{1}{2}$, $y = \frac{1}{4}$.

Common errors

- Dividing both sides by x and losing $x = 0$.
- Keeping a candidate from squaring without checking it.
- Forgetting the negative case of an absolute value.

Common errors (2)

- Keeping a solution that makes a denominator zero.
- Writing only the x-values when the question asks for points.

Key terms

- **discriminant**: The value $b^2 - 4ac$, which tells how many real solutions a quadratic has.
- **candidate root**: A possible solution that must still be checked in the original equation.
- **intersection**: A point that lies on both graphs.
- **parabola**: The U-shaped graph of a quadratic function.

Check yourself

- How many real solutions has $x^2 + 4x + 5 = 0$?
- Solve $|2x - 1| = 5$.
- Solve $x^2 = 9x$.