Equivalent expressions and domain restrictions

Original skill preparation; versioned source scope

Equivalent Expressions and Domain Restrictions

- Recognise **equivalent expressions** 等价表达式, equal for every allowed value.
- Expand brackets and **factor** 因式 expressions using standard patterns.
- State the **domain** 定义域 restrictions of a rational expression.
- Rewrite roots and powers correctly.

What "equivalent" means

- Two expressions are equivalent if they give the same value for every input in their domain.
- One matching test value does not prove equivalence: x^2 and $2x$ agree at $x = 2$ but not at $x = 3$.
- One different value does prove they are not equivalent.
- If two polynomials are equal for all x, their matching coefficients are equal. This lets you find unknown constants.

Expanding and factoring

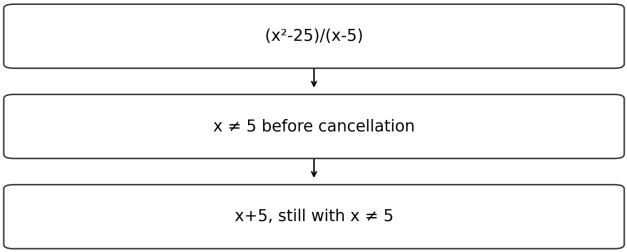
- Expand every term in one bracket by every term in the other.
- **Factor** by first taking out a common factor, then looking for a difference of squares or a trinomial.
- For $x^2 + bx + c$, find two numbers with product c and sum b.

$$(a+b)^2 = a^2 + 2ab + b^2 \qquad a^2 - b^2 = (a-b)(a+b)$$

Rational expressions and restrictions

- Find the restrictions from the original denominator before you cancel anything.
- Cancel only common factors, never common terms: $\frac{x+3}{x}$ is not 3.
- After cancelling, the restriction stays: $\frac{x^2-25}{x-5} = x+5$ only for $x \neq 5$.

Figure: Rational expressions and restrictions



- Worked steps: $(x^2\text{-}25)/(x\text{-}5) \rightarrow x \neq 5$ before cancellation $\rightarrow x+5$, still with $x \neq 5$
- Match each part of the diagram to the text.

Roots and powers

- $x^{1/n} = \sqrt[n]{x}$ and $x^{m/n} = (\sqrt[n]{x})^m$.
- $\sqrt{x^2} = |x|$, not x: at $x = -3$ it gives 3.
- $x^a \cdot x^b = x^{a+b}$ and $(x^a)^b = x^{ab}$, but $(x+y)^2 \neq x^2 + y^2$.

Worked example: a rational expression

- **Factor**: numerator $x(x-4)$, denominator $(x-4)(x+4)$.
- The original denominator is zero at $x=4$ and $x=-4$, so both are excluded.
- Cancel: $F(x)=\dfrac{x}{x+4}$ for $x\neq 4,-4$.
- $\dfrac{x}{x+4}$ alone allows $x=4$ (it gives $\frac{1}{2}$), but $F(4)$ is undefined, so they are not identical.

Worked example: matching coefficients

- Expand: $ax^2 - 2ax + 3x - 6 = ax^2 + (3 - 2a)x - 6$.
- Match x^2: $a = 4$.
- Match x: $b = 3 - 8 = -5$.
- Check at $x = 1$: left $(7)(-1) = -7$; right $4 - 5 - 6 = -7$.

Common errors

- Cancelling terms across a fraction bar instead of factors.
- Dropping a restriction after cancelling.
- Writing $(x+3)^2 = x^2 + 9$.

Common errors (2)

- Writing $\sqrt{x^2} = x$ for negative x.
- Treating one matching test value as proof.

Key terms

- **equivalent expressions**: Expressions with the same value for every allowed input.
- **factor**: An expression that multiplies with another to give a product; to **factor** is to write as such a product.
- **domain**: The set of input values for which an expression is defined.

Check yourself

- Expand $(x-3)(x+3)$.
- State the restrictions on $\dfrac{x+1}{x^2-9}$.
- Is $\sqrt{x^2} = x$ for every real x?