Geometry and Trigonometry

Original skill preparation; versioned source scope

Geometry and Trigonometry on the SAT

- Use area and volume formulas (most are given on the reference sheet).
- Use angle facts and **similarity** 相似, with its **scale factor** 比例因子.
- Use the Pythagorean theorem, special right triangles and trigonometric ratios.
- Work with circles: **arc length** 弧长, sectors, radians and equations.

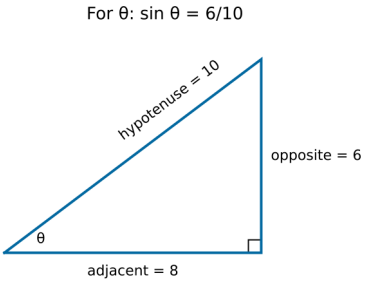
Lines, angles and triangles

- Angles on a straight line add to 180°; angles in a triangle add to 180°.
- Parallel lines cut by a transversal give equal corresponding and alternate angles.
- Similar triangles have equal angles and proportional sides. If lengths scale by k, areas scale by k^2 and volumes by k^3.

Right triangles

- Pythagorean theorem: $a^2 + b^2 = c^2$. Common triples: 3-4-5, 5-12-13, 8-15-17, and their multiples.
- Special triangles: 45-45-90 has sides $1 : 1 : \sqrt{2}$; 30-60-90 has sides $1 : \sqrt{3} : 2$.
- $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$.
- $\sin x^\circ = \cos(90 - x)^\circ$.

Figure: Right triangles



- sin θ = opposite $\div$ hypotenuse $= 6/10$
- Match each part of the diagram to the text.

Circles

- Circumference $2\pi r$, area πr^2.
- An arc with central angle θ degrees has length $\dfrac{\theta}{360} \times 2\pi r$, in radians, $s = r\theta$.
- $180° = \pi$ radians.
- The circle with centre (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$. Complete the square to find the centre and radius.

Worked example: similar triangles

- The area ratio is $216/96 = 9/4$, so the **scale factor** is $k = \sqrt{9/4} = 3/2$.
- A side of 8 in the smaller triangle matches $8 \times 1.5 = 12$ in the larger.
- In the 6-8-10 triangle of the figure, $\sin\theta = 0.6$, $\cos\theta = 0.8$ and $\tan\theta = 0.75$.

Worked example: a circle equation

- Complete the square: $(x-3)^2 - 9 + (y+2)^2 - 4 = 12$.
- So $(x-3)^2 + (y+2)^2 = 25$: centre $(3, -2)$, radius 5.
- A 72° arc of this circle has length $\dfrac{72}{360} \times 10\pi = 2\pi$.

Common errors

- Scaling area by k instead of k^2.
- Matching the wrong sides in similar triangles.
- Using the hypotenuse as "adjacent".

Common errors (2)

- Reading the centre of $(x-3)^2+(y+2)^2=25$ as $(-3,2)$.
- Giving r^2 as the radius.

Key terms

- **similarity**: Having the same shape: equal angles and proportional sides.
- **scale factor**: The number by which every length is multiplied in an enlargement.
- **arc length**: The distance along part of a circle's circumference.

Check yourself

- A right triangle has legs 5 and 12. What is the hypotenuse?
- If $\sin x^\circ = 0.4$, what is $\cos(90 - x)^\circ$?
- How many radians are in 135°?