Advanced Math

Original skill preparation; versioned source scope

Advanced Math on the SAT

- Rewrite **equivalent expressions** 等价表达式, keeping the **domain** 定义域 in mind.
- Solve quadratic equations and read each **root** 根, vertex and intercept from the right form.
- Model growth and decay with exponential functions.
- Use function notation and transformations of graphs.

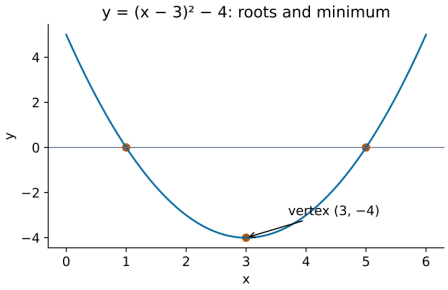
Equivalent expressions

- Expand, factor and simplify. Two expressions are equivalent if they are equal for every allowed input.
- Cancelling a factor can hide a restriction: $\dfrac{x^2-9}{x-3} = x+3$ only for $x \neq 3$.
- Common patterns: $a^2 - b^2 = (a-b)(a+b)$ and $(a+b)^2 = a^2 + 2ab + b^2$.

Quadratics

- Standard form $ax^2 + bx + c$: the y-intercept is c.
- Factored form $a(x - r)(x - s)$: the roots (x-intercepts) are r and s.
- Vertex form $a(x - h)^2 + k$: the vertex is (h, k), a minimum if $a > 0$, a maximum if $a < 0$.
- Quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The discriminant $b^2 - 4ac$ gives two real roots if positive, one if zero, none if negative.

Figure: Quadratics



- Vertex form shows the minimum $(3, -4)$; setting $y = 0$ gives the roots 1 and 5
- Match each part of the diagram to the text.

Exponential functions

- $f(t) = a(1 + r)^t$ grows by the same percentage r each period; $a(1 - r)^t$ decays.
- a is the starting value, and $1 + r$ is the growth factor: 1.05 means 5% growth per period.
- Linear models add the same amount each step; exponential models multiply by the same factor.

Functions and transformations

- $f(x) + k$ moves the graph up by k; $f(x - h)$ moves it right by h.
- $-f(x)$ reflects it in the x-axis; $af(x)$ stretches it vertically by a.
- To evaluate $f(g(2))$, find $g(2)$ first.

Worked example: a parabola

- Vertex form: the minimum is at $(3, -4)$.
- Roots: $(x-3)^2 = 4$, so $x - 3 = \pm 2$ and $x = 1$ or 5.
- Standard form: $x^2 - 6x + 5$, so the y-intercept is 5. Discriminant $36 - 20 = 16 > 0$: two roots, as found.

Worked example: growth

- $P(t) = 20000(1.03)^t$.
- After 10 years: $20000 \times 1.03^{10} = 20000 \times 1.344 = 26,900$ (to 3 significant figures).
- A linear model adding 600 a year would give only 26,000, and the gap widens each year.

Common errors

- Cancelling a factor and forgetting the excluded value.
- Reading the vertex as $(-h, k)$ from $(x-h)^2 + k$.
- Treating 5% growth as a factor of 0.05 instead of 1.05.

Common errors (2)

- Moving a graph left for $f(x-h)$.
- Confusing a **root** with the y-intercept.

Key terms

- **equivalent expressions**: Expressions that are equal for every allowed value of the variable.
- **domain**: The set of input values for which a function or expression is defined.
- **root**: A value of x that makes a function equal to zero.

Check yourself

- How many real roots does $x^2 + 2x + 5 = 0$ have?
- A car worth 24,000 dollars loses 15% of its value each year. Write its value after t years.
- What is the vertex of $y = -2(x+1)^2 + 7$?