Algebra

Original skill preparation; versioned source scope

Algebra on the SAT

- Solve linear equations and inequalities, and count their solutions.
- Read the **slope** 斜率 and **intercept** 截距 of a linear model in context.
- Solve systems of two linear equations.
- Write an equation or inequality from words.

Linear equations

- Collect the variable on one side: $ax + b = cx + d$ becomes $(a - c)x = d - b$.
- If $a \neq c$, there is one solution, $x = \dfrac{d - b}{a - c}$.
- If $a = c$ and $b = d$, every number works (infinitely many solutions). If $a = c$ but $b \neq d$, no number works.
- Check the coefficient before dividing by it.

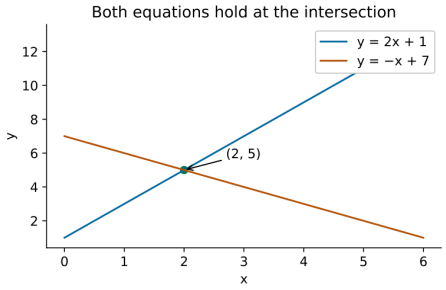
Slope and intercept in context

- The slope m is the change in y for each one-unit increase in x, with units "output per input".
- The **intercept** b is the value of y when $x = 0$, often a starting amount or fixed fee.
- The intercept means nothing real if $x = 0$ lies outside the situation.

Systems of equations

- Substitution: solve one equation for a variable and put it into the other.
- Elimination: add or subtract multiples of the equations to remove a variable.
- Parallel lines (same slope, different intercepts): no solution. The same line: infinitely many.

Figure: Systems of equations



- The intersection is the one point that satisfies both equations
- Match each part of the diagram to the text.

Inequalities

- Solve like an equation, but reverse the sign when multiplying or dividing by a negative number.
- On a graph, a solid boundary means "or equal to"; a dashed one means strict.
- Test one point to check which side is shaded.

Worked example: a fare

- $f + 3r = 11$ and $f + 7r = 19$. Subtracting: $4r = 8$, so $r = 2$.
- Then $f = 11 - 3 \times 2 = 5$. The slope, 2 dollars per km, is the rate; the **intercept**, 5 dollars, is the fixed fee.
- With 40 dollars, $5 + 2k \le 40$ gives $k \le 17.5$: at most 17 whole kilometres.

Worked example: counting solutions

- Collect terms: $(p-3)x = p-3$.
- If $p = 3$: $0 = 0$, so every real x works.
- If $p \neq 3$: $x = 1$, one solution. No value of p gives no solution. Check: $x = 1$ gives $p + 2$ on both sides.

Common errors

- Dividing by an expression that might be zero.
- Forgetting to reverse an inequality when dividing by a negative.
- Reading the slope as the starting value, or the reverse.

Common errors (2)

- Treating a system with the same slope as having one solution.
- Rounding up a whole-number answer that must stay within a budget.

Key terms

- **slope**: The change in output for each one-unit increase in input.
- **intercept**: The output when the input is zero; often a starting amount.

Check yourself

- Solve $3x - 7 = 2x + 5$.
- How many solutions does the system $2x + 3y = 6$, $4x + 6y = 12$ have?
- A plant is $h = 1.5w + 4$ cm tall after w weeks. What does 1.5 mean?