Course Companion

SAT

Every topic handout in one volume

全部专题讲义合集

exercises.lol

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Information and Ideas

SAT

Information and Ideas on the SAT

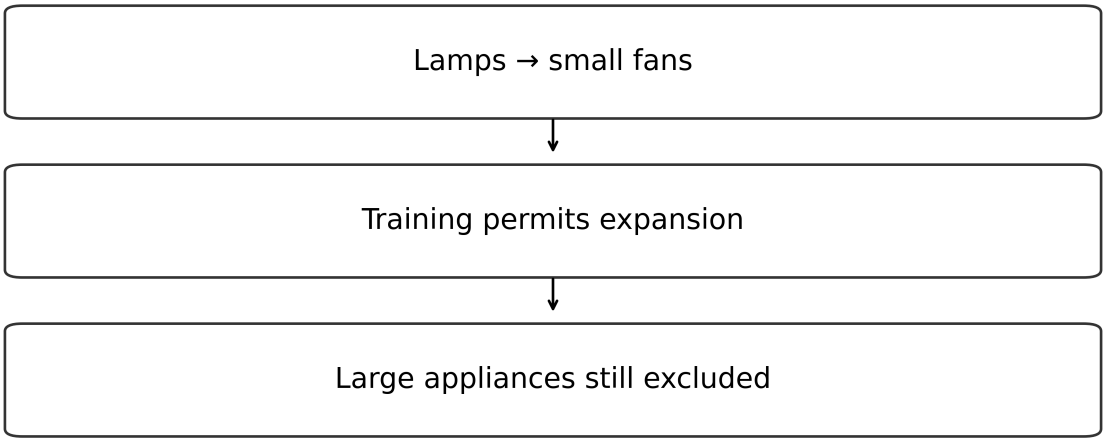
In 1847 Ignaz Semmelweis, a doctor at Vienna General Hospital, compared two maternity clinics. In the clinic run by doctors, who often came straight from dissecting bodies, far more mothers died of fever than in the clinic run by midwives. In May 1847 he made the doctors wash their hands in chlorine solution, and deaths in their clinic fell to about the midwives' level within months. His data supported a precise claim: handwashing in that clinic cut deaths. Many colleagues rejected it, partly because he claimed more than he had shown about the cause. Reading evidence exactly, and claiming only what it supports, is what this SAT domain tests.

On the Digital SAT, Reading and Writing has 54 questions in 64 minutes, in two modules. Each question has its own short passage. Information and Ideas questions (about a quarter of the section) ask you to:

- State the **central idea** 中心思想 of a passage and tell it apart from a **detail** 细节.
- Choose the **evidence** 证据, a quotation or data, that supports a claim.
- Read tables and graphs accurately.
- Make the **inference** 推断 that the text allows, and no more.

Central ideas and details

- The central idea is the point the whole passage makes, usually a relationship: a change, a contrast, a cause with limits.
- A detail supports the central idea but is not it.
- Summarise each sentence in a few words, then ask what joins them.
- Wrong answers are often too broad ("it repaired everything") or too narrow (one detail only).



Worked steps: Lamps → small fans → Training permits expansion → Large appliances still excluded

Textual and quantitative evidence

- First restate the claim in your own words. Then test each option: does it make the claim more likely?
- An option can be true and interesting but not support this claim.
- For a table or graph, read the title, labels, units and groups before the numbers.
- Compare only the groups the claim names, and only within the data shown.

Inferences

- A good inference joins information the passage states. It adds no new cause, group or time.
- Check every word of the answer: “always”, “all” or “proves” usually go further than the text.
- The right answer often sounds modest.

Worked example: a central idea

“The repair café first accepted only lamps. It later added small fans after volunteers received training. Large appliances still went to specialist shops.”

- Summaries: started with lamps; expanded after training; limits remain.
- Central idea: the café expanded its repairs, within limits.
- “It repaired everything” ignores the last sentence; “It repaired lamps” leaves out the change.

Worked example: data and scope

A study measured two streets at noon on one day: the shaded street was 29 °C, the unshaded street 33 °C.

- Supported: at those sites and that time, the shaded street was 4 °C cooler.
- Not supported: trees cool the whole city, or shade caused the whole difference, since the streets may differ in other ways.
- The answer that keeps the place, the time and the size of the effect is the strongest.

Common errors

- Choosing a vivid detail as the central idea.
- Choosing evidence that is true but supports a different claim.
- Reading a graph's numbers before its labels and units.
- Extending a finding to new groups, places or times.
- Treating a correlation in the passage as a proven cause.

Key terms

central idea: The main point that the whole passage makes.

detail: A specific fact or example that supports the central idea.

evidence: Information from the text or data that makes a claim more likely to be true.

inference: A conclusion that follows from what the text states.

Check yourself

- A passage says a library's new index "helped readers find likely pages but did not replace reading the registers." What is its central idea?
- A claim says sales rose in 2021. A table shows 2019 and 2020 only. Can it support the claim?
- Why is "proves" a warning word in an answer choice?

Answers: the index is a useful starting point with limits. No: the data do not reach 2021. It usually claims more than the passage supports.

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Craft and Structure

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Craft and Structure on the SAT

In 1879 James Murray, a Scottish schoolteacher, took charge of a new dictionary that would record every English word with dated quotations showing how its meaning had changed. The first part of the *Oxford English Dictionary* appeared in 1884; the last in 1928, after Murray's death. Volunteers sent in millions of slips of paper with quotations. The work showed how much one word's meaning depends on its context: by the 1989 second edition, the verb "set" needed more than 400 senses. SAT Craft and Structure questions test that same awareness of context, along with how texts are built.

Craft and Structure questions (about 28% of Reading and Writing) ask you to:

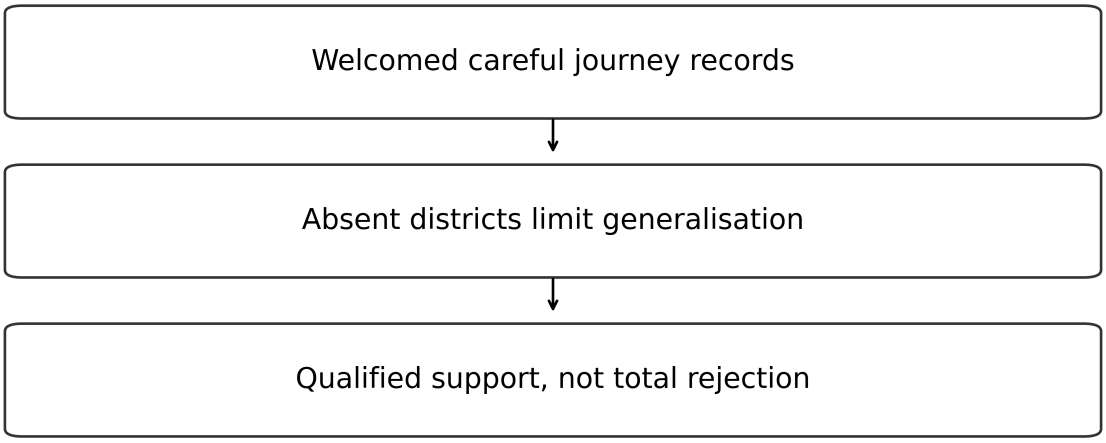
- Choose the meaning of a **word in context** 语境中的词义.
- Describe the structure and purpose of a text.
- Compare two texts, including where they agree and where they show **contrast** 对比.
- Use **evidence** 证据 from the passage for each choice.

Words in context

- Cover the word and predict what the sentence needs, using clues nearby: examples, contrasts, causes.
- Then choose the option that matches your prediction in meaning and tone (positive, negative or neutral).
- The most common meaning is often a trap. In "The results were qualified", qualified means limited, not "having qualifications".

Structure and purpose

- Ask what each sentence does: introduce a claim, give an example, describe a problem, present an objection, answer it.
- The purpose of the whole text is often a verb: to explain, to challenge, to compare, to describe a finding and its limits.
- Words like "however", "yet" and "but" mark a turn; the main point often follows them.



Worked steps: Welcomed careful journey records → Absent districts limit generalisation → Qualified support, not total rejection

Comparing two texts

- Summarise each text's claim separately before comparing.
- Ask how the second author would respond to the first: agree, disagree, or agree with a limit.
- Do not merge the texts into one view, or give one author's reason to the other.

Worked example: three words

A reviewer "welcomed the careful journey records" but had a "reservation": some districts were missing, so the findings were "qualified" and needed wider sampling. Even so, they were "illuminating".

- Reservation: a doubt or limit (clue: the missing districts), not a booked seat.
- Qualified: limited or conditional (clue: the need for wider sampling).
- Illuminating: making something clearer (clue: the positive "even so").

Worked example: two texts

Text 1: a city's new bike lanes reduced car traffic downtown. Text 2: traffic fell, but the same year a new train line opened, so the lanes' share of the drop is unclear.

- Text 2 accepts the finding (traffic fell) but questions the cause.
- So the second author would respond: "The drop may have more than one cause."
- An answer saying the second author "rejects the data" goes too far.

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Common errors

- Choosing the most familiar meaning of a word instead of the one the context needs.
- Missing the turn at “however” or “yet”.
- Describing a text’s topic when the question asks for its purpose.
- Treating partial agreement as total disagreement.
- Mixing up which author said what.

Key terms

word in context: The meaning a word has in a particular sentence.

contrast: A difference between two ideas, often signalled by words like “but” or “however”.

evidence: Words or data from the text that support an answer.

Check yourself

- In “The committee tabled the proposal until spring”, what does “tabled” mean?
- A passage describes a theory, then a study that challenged it. What is the function of the second part?
- Text 2 says Text 1’s findings “hold only for small towns”. Is that agreement, disagreement or partial agreement?

Answers: postponed. To present a challenge to the theory. Partial agreement.

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Expression of Ideas

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Expression of Ideas on the SAT

In 1918 William Strunk Jr., an English professor at Cornell University, printed a small book for his students. Rule 17 said: "Omit needless words." One of those students, E. B. White, later the author of *Charlotte's Web*, revised and expanded it in 1959. *The Elements of Style* has sold millions of copies. SAT Expression of Ideas questions reward the same habits: choose the words that do the job asked, and connect ideas with the right logical link.

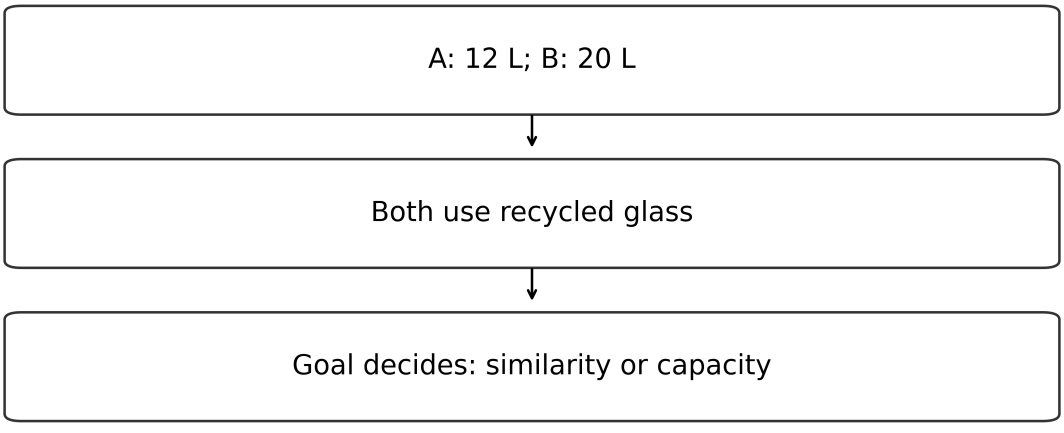
Expression of Ideas questions (about 20% of Reading and Writing) ask you to:

- Use **rhetorical synthesis** 修辞综合: choose information from a writer's notes that meets a stated goal.
- Choose the **transition** 衔接词 that shows the right relationship between two ideas.
- Recognise a **contrast marker** 转折标记 and a word that shows **similarity** 相似.

Rhetorical synthesis

These questions give bullet-point notes and a goal, such as "emphasise a similarity" or "introduce the study to an audience unfamiliar with it".

- Read the goal first, and underline its key words.
- Choose the option that does exactly that, using only facts from the notes.
- An option can be accurate and well written but meet a different goal.
- **Synthesis** 综合 here means combining notes, not adding new claims.



Worked steps: A: 12 L; B: 20 L → Both use recycled glass → Goal decides: similarity or capacity

Transitions

Decide the relationship between the two sentences before reading the options.

- Addition: furthermore, in addition, also.
- Contrast: however, nevertheless, by contrast, yet.
- Cause and result: therefore, as a result, consequently.
- Example: for example, for instance. Sequence: then, finally, subsequently.
- Similarity: likewise, similarly.

Choosing precisely

- Keep the meaning of the notes; do not add a cause, a number or an opinion they do not contain.
- A shorter, clearer option is usually better if it says the same.
- Read the sentence before and after: the transition must fit both.

Worked example: two goals

Notes: Bay Park and Hill Park used volunteer teams. Bay restored 3 km of paths; Hill restored 5 km.

- Goal: emphasise a similarity. "Both Bay and Hill used volunteer teams."
- Goal: emphasise a difference in distance. "Hill restored 5 km of paths, 2 km more than Bay."
- "Hill's volunteers worked harder" adds a cause the notes do not give.

Worked example: a transition

"The new glass containers hold more than the old ones. _____, they weigh less."

- Both sentences are advantages, so the link is addition.
- "Moreover" or "In addition" fits. "However" would signal a contrast that is not there.
- If the second sentence were "they cost more", the link would be contrast: "However".

Common errors

- Choosing an accurate option that does not meet the stated goal.
- Adding information that is not in the notes.
- Choosing a transition by its sound, before deciding the relationship.
- Using "therefore" when the second idea is not a result of the first.
- Ignoring the sentence after the blank.

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Key terms

rhetorical synthesis: Choosing and combining information from notes to achieve a stated purpose.

transition: A word or phrase that shows how two ideas are related.

contrast marker: A transition that signals a difference, such as "however".

similarity: Something two things have in common.

synthesis: Combining information from several sources into one statement.

Check yourself

- "The plan was cheap. _____, it was popular." Which transition fits?
- "The bridge was closed for repairs. _____, traffic moved to the ring road." Which kind of relationship is this?
- A goal says "present the study's main finding". Should the answer describe its method?

Answers: "Moreover" (addition). Cause and result ("As a result"). No: it should state the finding.

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Standard English Conventions

SAT

Standard English Conventions on the SAT

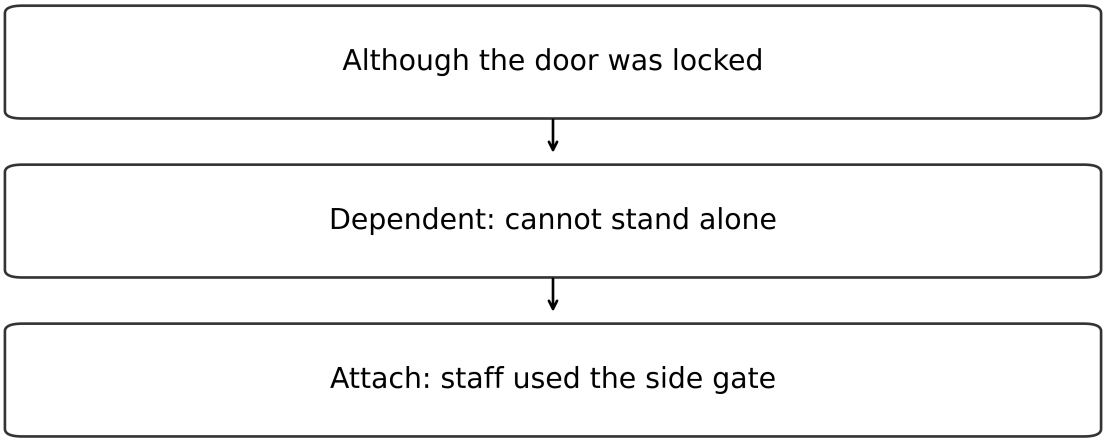
In 2003 Lynne Truss, a British journalist, published a book about punctuation with a joke for its title. A panda walks into a café, eats a sandwich, fires a gun and leaves. When the waiter asks why, the panda points to a badly punctuated wildlife guide: "Panda. Large black-and-white bear-like mammal. Eats, shoots and leaves." *Eats, Shoots & Leaves* sold millions of copies. The comma turned "shoots" and "leaves", the nouns for bamboo, into verbs. SAT Conventions questions are about the rules that keep meaning clear.

Standard English Conventions questions (about a quarter of Reading and Writing) ask you to:

- Mark each **sentence boundary** 句子边界 correctly.
- Use commas, a **colon** 冒号, semicolons and dashes with complete and incomplete **clauses** 分句.
- Check subject–verb agreement, pronouns and verb forms.
- Place modifiers next to what they describe.

Clauses

- An independent clause has a subject and a **finite verb** 限定动词 and can stand alone: "The door was locked."
- A dependent clause cannot: "Although the door was locked".
- A dependent clause before the main clause is followed by a comma: "Although the door was locked, staff used the side gate."



Worked steps: Although the door was locked → Dependent: cannot stand alone → Attach: staff used the side gate

Joining independent clauses

Two independent clauses can be joined by:

- A full stop, or a semicolon.
- A comma plus a coordinating conjunction: and, but, or, so, yet, for, nor.
- Not by a comma alone (a comma splice), and not by nothing (a run-on).
- "However" is not a conjunction: write "; however," or ". However,".

Colons, dashes and nonessential information

- A colon must follow a complete clause. It introduces a list, an explanation or an example: "One part needed replacement: the hinge."
- Nonessential information is marked with a matching pair: two commas, two dashes or two brackets. Do not mix them.
- Essential information that identifies which one is meant takes no commas: "The student who won spoke first."

Agreement and form

- A verb agrees with its subject, not with a noun just before it: "The box of tools is heavy."
- A pronoun agrees with the noun it replaces: "The committee gave its report."
- A modifier at the start of a sentence describes the subject that follows: "Walking home, she saw…", not "Walking home, the rain…".

Worked example: punctuation in a paragraph

"After the pump failed [1] engineers found the cause [2] they repaired it in a day. One part needed replacement [3] the valve."

1 Comma: an introductory dependent clause.

2 Semicolon or full stop: two independent clauses. A comma alone would be a splice.

3 Colon: a complete clause introduces the item.

Worked example: agreement

"The list of approved suppliers ____ updated every month."

- The subject is "list", singular; "of approved suppliers" is a phrase in between.
- So the answer is "is", not "are".
- Test by removing the phrase: "The list is updated every month."

Common errors

- Joining two independent clauses with a comma alone.
- Using a colon after an incomplete clause ("The items are: …").
- Opening nonessential information with a comma and closing it with a dash.
- Matching the verb to the nearest noun instead of the subject.
- Leaving a modifier with nothing it can describe.

Key terms

sentence boundary: The point where one complete sentence ends and the next begins.

finite verb: A verb with tense that can act as the main verb of a clause.

clauses: Groups of words with a subject and a verb.

colon: A punctuation mark (:) that follows a complete clause and introduces what explains or lists it.

Check yourself

- Correct: "The test was hard, most students finished it."
- Choose: "Each of the players ____ a number." (has / have)
- Is this colon correct: "We need: flour, eggs and milk."?

Answers: "hard; most" or "hard, but most". Has. No: "need" does not complete the clause; remove the colon.

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Algebra

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Algebra on the SAT

Around 820 the scholar Muhammad ibn Musa al-Khwarizmi, working in Baghdad's House of Wisdom, wrote a book on solving equations by two moves: *al-jabr*, "restoring" a subtracted term by adding it to both sides, and *al-muqabala*, "balancing" equal terms on each side. Translated into Latin, *al-jabr* became "algebra". The SAT's Algebra questions use exactly those moves, on linear equations, inequalities and systems, and add one more skill: reading what each number means in a real situation.

SAT Algebra questions ask you to:

- Solve linear equations and inequalities, and count their solutions.
- Read the **slope** 斜率 and **intercept** 截距 of a linear model in context.
- Solve systems of two linear equations.
- Write an equation or inequality from words.

Linear equations

- Collect the variable on one side: $ax + b = cx + d$ becomes $(a - c)x = d - b$.
- If $a \neq c$, there is one solution, $x = \dfrac{d - b}{a - c}$.
- If $a = c$ and $b = d$, every number works (infinitely many solutions). If $a = c$ but $b \neq d$, no number works.
- Check the coefficient before dividing by it.

Slope and intercept in context

In $y = mx + b$:

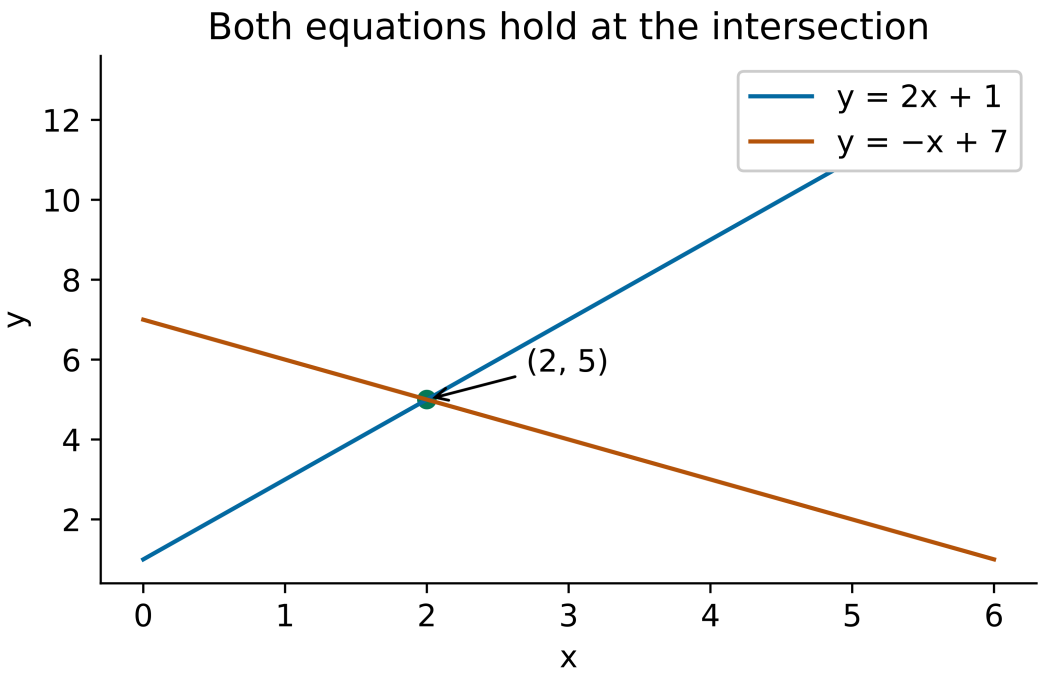
- The slope m is the change in y for each one-unit increase in x, with units "output per input".
- The intercept b is the value of y when $x = 0$, often a starting amount or fixed fee.
- The intercept means nothing real if $x = 0$ lies outside the situation.

Systems of equations

A solution of a system satisfies every equation at once: on a graph, it is where the lines cross.

- Substitution: solve one equation for a variable and put it into the other.
- Elimination: add or subtract multiples of the equations to remove a variable.

- Parallel lines (same slope, different intercepts): no solution. The same line: infinitely many.



The intersection is the one point that satisfies both equations

Inequalities

- Solve like an equation, but reverse the sign when multiplying or dividing by a negative number.
- On a graph, a solid boundary means "or equal to"; a dashed one means strict.
- Test one point to check which side is shaded.

Worked example: a fare

A taxi charges a fixed fee f plus r per kilometre. A 3 km ride costs 11 dollars and a 7 km ride 19 dollars.

- $f + 3r = 11$ and $f + 7r = 19$. Subtracting: $4r = 8$, so $r = 2$.
- Then $f = 11 - 3 \times 2 = 5$. The slope, 2 dollars per km, is the rate; the intercept, 5 dollars, is the fixed fee.
- With 40 dollars, $5 + 2k \le 40$ gives $k \le 17.5$: at most 17 whole kilometres.

Worked example: counting solutions

For which p does $(p-1)x+3=2x+p$ have no solution, one, or infinitely many?

- Collect terms: $(p-3)x=p-3$.
- If $p=3$: $0=0$, so every real x works.
- If $p\neq 3$: $x=1$, one solution. No value of p gives no solution. Check: $x=1$ gives $p+2$ on both sides.

Common errors

- Dividing by an expression that might be zero.
- Forgetting to reverse an inequality when dividing by a negative.
- Reading the slope as the starting value, or the reverse.
- Treating a system with the same slope as having one solution.
- Rounding up a whole-number answer that must stay within a budget.

Key terms

slope: The change in output for each one-unit increase in input.

intercept: The output when the input is zero; often a starting amount.

Check yourself

- Solve $3x-7=2x+5$.
- How many solutions does the system $2x+3y=6$, $4x+6y=12$ have?
- A plant is $h=1.5w+4$ cm tall after w weeks. What does 1.5 mean?

Answers: $x=12$. Infinitely many (the same line). It grows 1.5 cm per week.

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Advanced Math

SAT

Advanced Math on the SAT

In 1638 Galileo Galilei published *Two New Sciences*, written while he was under house arrest. In it he showed, by rolling balls down a gentle slope and timing them with a water clock, that the distance a falling body covers grows with the square of the time: in twice the time, four times the distance. It was one of the first laws of nature written as a nonlinear function. The SAT's Advanced Math questions are about such functions: quadratics, exponentials and others that are not straight lines.

SAT Advanced Math questions ask you to:

- Rewrite **equivalent expressions** 等价表达式, keeping the **domain** 定义域 in mind.
- Solve quadratic equations and read each **root** 根, vertex and intercept from the right form.
- Model growth and decay with exponential functions.
- Use function notation and transformations of graphs.

Equivalent expressions

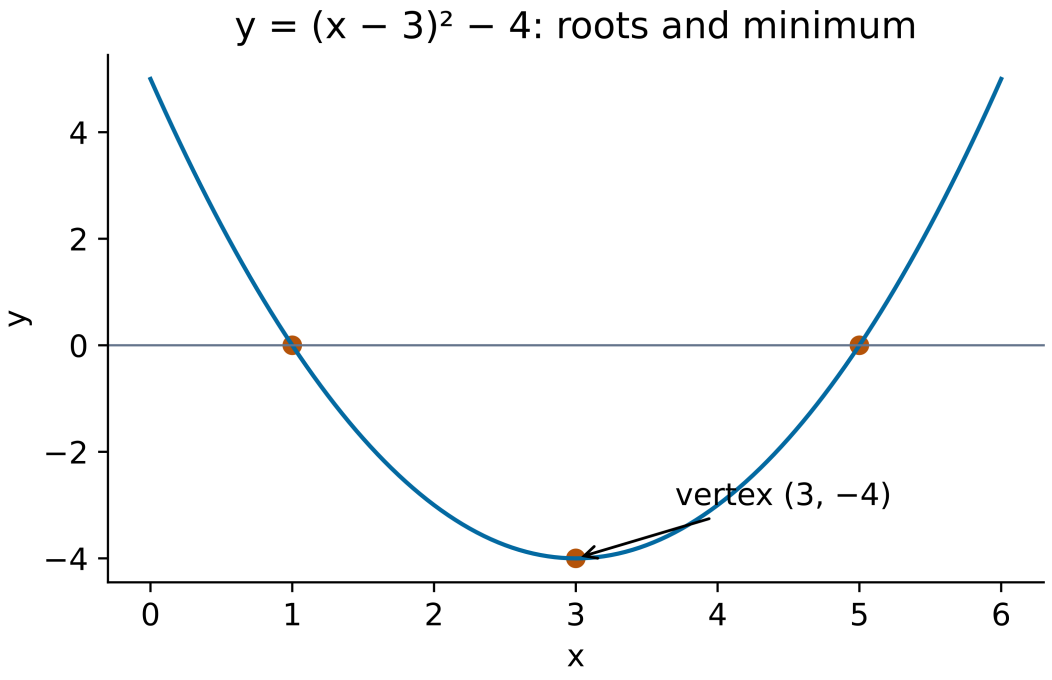
- Expand, factor and simplify. Two expressions are equivalent if they are equal for every allowed input.
- Cancelling a factor can hide a restriction: $\dfrac{x^2-9}{x-3} = x+3$ only for $x \neq 3$.
- Common patterns: $a^2 - b^2 = (a-b)(a+b)$ and $(a+b)^2 = a^2 + 2ab + b^2$.

Quadratics

Each form shows something different:

- Standard form $ax^2 + bx + c$: the y-intercept is c.
- Factored form $a(x-r)(x-s)$: the roots (x-intercepts) are r and s.
- Vertex form $a(x-h)^2 + k$: the vertex is (h, k), a minimum if $a > 0$, a maximum if $a < 0$.
- Quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The discriminant $b^2 - 4ac$ gives two real roots if positive, one if zero, none if negative.

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Vertex form shows the minimum (3, −4); setting $y = 0$ gives the roots 1 and 5

Exponential functions

- $f(t) = a(1 + r)^t$ grows by the same percentage r each period; $a(1 - r)^t$ decays.
- a is the starting value, and $1 + r$ is the growth factor: 1.05 means 5% growth per period.
- Linear models add the same amount each step; exponential models multiply by the same factor.

Functions and transformations

- $f(x) + k$ moves the graph up by k; $f(x - h)$ moves it right by h.
- $-f(x)$ reflects it in the x-axis; $af(x)$ stretches it vertically by a.
- To evaluate $f(g(2))$, find $g(2)$ first.

Worked example: a parabola

$y = (x - 3)^2 - 4$.

- Vertex form: the minimum is at $(3, -4)$.
- Roots: $(x - 3)^2 = 4$, so $x - 3 = \pm 2$ and $x = 1$ or 5.
- Standard form: $x^2 - 6x + 5$, so the y-intercept is 5. Discriminant $36 - 20 = 16 > 0$: two roots, as found.

Worked example: growth

A town of 20,000 grows 3% a year.

- $P(t) = 20000(1.03)^t$.
- After 10 years: $20000 \times 1.03^{10} = 20000 \times 1.344 = 26,900$ (to 3 significant figures).
- A linear model adding 600 a year would give only 26,000, and the gap widens each year.

Common errors

- Cancelling a factor and forgetting the excluded value.
- Reading the vertex as $(-h, k)$ from $(x - h)^2 + k$.
- Treating 5% growth as a factor of 0.05 instead of 1.05.
- Moving a graph left for $f(x - h)$.
- Confusing a root with the y-intercept.

Key terms

equivalent expressions: Expressions that are equal for every allowed value of the variable.

domain: The set of input values for which a function or expression is defined.

root: A value of x that makes a function equal to zero.

Check yourself

- How many real roots does $x^2 + 2x + 5 = 0$ have?
- A car worth 24,000 dollars loses 15% of its value each year. Write its value after t years.
- What is the vertex of $y = -2(x + 1)^2 + 7$?

Answers: none (discriminant -16). $V = 24000(0.85)^t$. $(-1, 7)$, a maximum.

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Problem-Solving and Data Analysis

SAT

Problem-Solving and Data Analysis on the SAT

In 1936 the magazine *Literary Digest* mailed questionnaires to about ten million Americans, chosen from telephone directories and car registrations, and received 2.4 million replies. It predicted a clear win for Alf Landon over President Franklin Roosevelt. Roosevelt won 46 of the 48 states. George Gallup, polling only about 50,000 people chosen to represent the country, called the result correctly. Owning a telephone or a car in 1936 meant being better off than average, and the poorer voters left out backed Roosevelt. A huge sample cannot repair a biased one.

SAT Problem-Solving and Data Analysis questions ask you to:

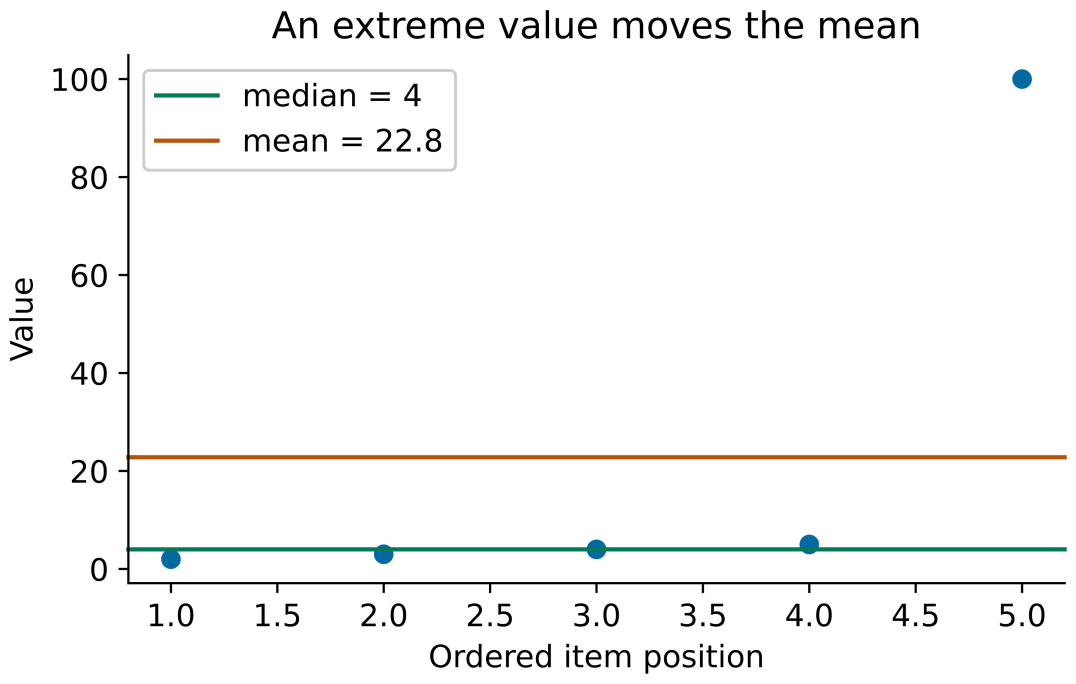
- Work with ratios, a **rate** 比率, percentages and unit conversions.
- Describe data with the mean, **median** 中位数 and spread.
- Read scatterplots and lines of best fit.
- Find probabilities from tables.
- Judge what a **random sample** 随机样本 or an experiment allows you to conclude.

Ratios, rates and percentages

- A rate compares two quantities with different units: 60 km per hour, 3 dollars per kg.
- Convert units by multiplying by fractions equal to 1: $\dfrac{1000\text{ m}}{1\text{ km}}$.
- Percentage change $= \dfrac{\text{new} - \text{old}}{\text{old}} \times 100\%$.
- Successive changes multiply: up 20% then down 20% is $1.2 \times 0.8 = 0.96$, a 4% fall overall.

Centre and spread

- The mean is pulled toward extreme values; the median is not.
- The range and standard deviation measure spread. Adding the same number to every value changes the mean but not the spread.
- In a skewed distribution with a long right tail, the mean is greater than the median.



One extreme value pulls the mean up to 22.8 but leaves the median at 4

Scatterplots and models

- A line of best fit summarises a linear trend; its slope is the predicted change in y per unit of x.
- A residual is actual minus predicted.
- Predicting far outside the data's range (extrapolation) is risky.

Probability and tables

- Probability = favourable outcomes ÷ possible outcomes.
- In a two-way table, "given" restricts you to one row or column: divide by that row's or column's total.

Samples and experiments

- A random sample lets you generalise to the population it was drawn from, with a margin of error.
- Random assignment to treatments lets you conclude cause and effect.
- Without random assignment, an association does not show cause.

Worked example: mean and median

The values are 2, 3, 4, 5 and 100.

- Median: the middle value, 4.
- Mean: $114/5 = 22.8$.
- Removing the 100 changes the mean to 3.5 and the median to 3.5. The median was the better summary of a typical value.

Worked example: a table

In a survey of 200 students, 120 play a sport; 45 of those also play an instrument, and 35 of the 80 non-players play an instrument.

- $P(\text{instrument}) = (45 + 35)/200 = 0.40$.
- $P(\text{instrument} \mid \text{sport}) = 45/120 = 0.375$.
- $P(\text{sport} \mid \text{instrument}) = 45/80 = 0.5625$. The "given" group sets the denominator.

Common errors

- Adding successive percentages instead of multiplying.
- Using the mean to describe a skewed set of data.
- Dividing by the whole table when the question says "given".
- Concluding cause from a survey.
- Generalising from a sample to a population it was not drawn from.

Key terms

rate: A ratio comparing quantities with different units.

median: The middle value of an ordered list of data.

random sample: A sample in which every member of the population has a known chance of being chosen.

Check yourself

- A price rises 10% and then 10% again. What is the total percentage increase?
- A data set has mean 50 and SD 6. Every value is increased by 4. What are the new mean and SD?
- A study randomly assigns volunteers to two diets. Can it show the diet caused a difference?

Answers: 21%. 54 and 6. Yes, for people like the volunteers.

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Geometry and Trigonometry

SAT

Geometry and Trigonometry on the SAT

Ancient writers tell a story about Thales of Miletus, around 600 BC. Visiting Egypt, he measured the height of a pyramid without climbing it. He waited for the time of day when his own shadow was as long as he was tall; at that moment, the pyramid's shadow (measured from its centre) was as long as the pyramid was high. The story may be a legend, but the method is sound: the sun's rays make the same angle with every vertical object, so the triangles are similar. SAT Geometry questions are built on the same few ideas.

SAT Geometry and Trigonometry questions ask you to:

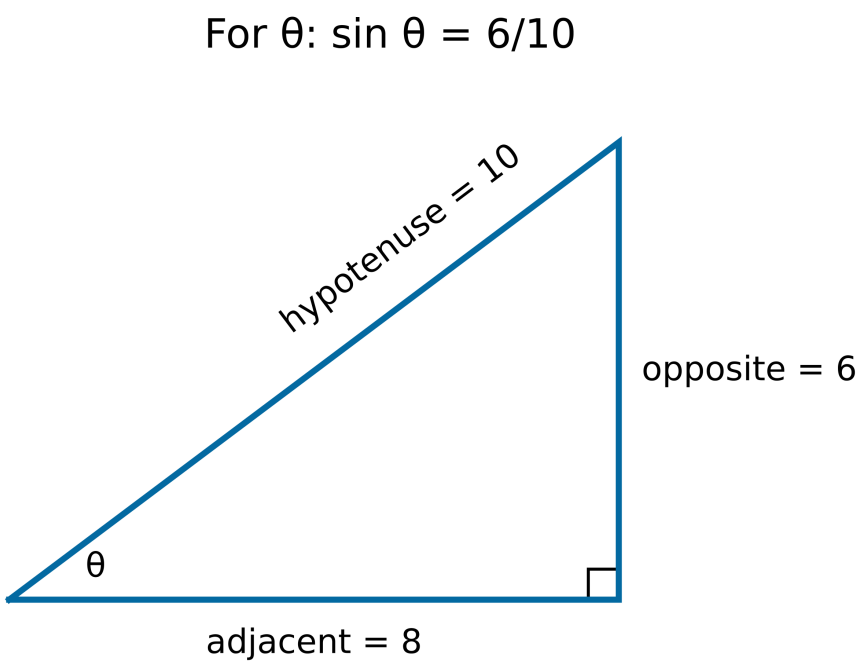
- Use area and volume formulas (most are given on the reference sheet).
- Use angle facts and **similarity** 相似, with its **scale factor** 比例因子.
- Use the Pythagorean theorem, special right triangles and trigonometric ratios.
- Work with circles: **arc length** 弧长, sectors, radians and equations.

Lines, angles and triangles

- Angles on a straight line add to 180°; angles in a triangle add to 180°.
- Parallel lines cut by a transversal give equal corresponding and alternate angles.
- Similar triangles have equal angles and proportional sides. If lengths scale by k, areas scale by k^2 and volumes by k^3.

Right triangles

- Pythagorean theorem: $a^2 + b^2 = c^2$. Common triples: 3-4-5, 5-12-13, 8-15-17, and their multiples.
- Special triangles: 45-45-90 has sides $1 : 1 : \sqrt{2}$; 30-60-90 has sides $1 : \sqrt{3} : 2$.
- $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$.
- $\sin x^\circ = \cos(90 - x)^\circ$.



sin θ = opposite ÷ hypotenuse = 6/10

Circles

- Circumference $2\pi r$, area πr^2.
- An arc with central angle θ degrees has length $\dfrac{\theta}{360} \times 2\pi r$; in radians, $s = r\theta$.
- $180° = \pi$ radians.
- The circle with centre (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$. Complete the square to find the centre and radius.

Worked example: similar triangles

Two similar triangles have areas 96 and 216.

- The area ratio is $216/96 = 9/4$, so the scale factor is $k = \sqrt{9/4} = 3/2$.
- A side of 8 in the smaller triangle matches $8 \times 1.5 = 12$ in the larger.
- In the 6-8-10 triangle of the figure, $\sin\theta = 0.6$, $\cos\theta = 0.8$ and $\tan\theta = 0.75$.

Worked example: a circle equation

Find the centre and radius of $x^2 - 6x + y^2 + 4y = 12$.

- Complete the square: $(x-3)^2 - 9 + (y+2)^2 - 4 = 12$.
- So $(x-3)^2 + (y+2)^2 = 25$: centre $(3, -2)$, radius 5.
- A 72° arc of this circle has length $\dfrac{72}{360} \times 10\pi = 2\pi$.

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Common errors

- Scaling area by k instead of k^2.
- Matching the wrong sides in similar triangles.
- Using the hypotenuse as "adjacent".
- Reading the centre of $(x-3)^2+(y+2)^2=25$ as $(-3,2)$.
- Giving r^2 as the radius.

Key terms

similarity: Having the same shape: equal angles and proportional sides.

scale factor: The number by which every length is multiplied in an enlargement.

arc length: The distance along part of a circle's circumference.

Check yourself

- A right triangle has legs 5 and 12. What is the hypotenuse?
- If $\sin x^\circ = 0.4$, what is $\cos(90-x)^\circ$?
- How many radians are in 135°?

Answers: 13. 0.4. $3\pi/4$.

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Linear parameters, solution counts and inequalities

SAT

Linear Equations: Parameters, Solution Counts and Inequalities

In 1557 the Welsh mathematician Robert Recorde published *The Whetstone of Witte* and introduced the equals sign. He chose two parallel lines, he wrote, "bicause noe 2 thynges can be moare equalle." Parallel lines are also the picture behind one of the SAT's favourite questions: two linear equations whose graphs are parallel never meet, so the system has no solution. This handout goes beyond the Algebra overview. It treats equations with a letter in place of a number, and the regions that inequalities describe.

SAT Algebra questions at this level ask you to:

- Decide whether a linear equation with a **parameter** 参数 has no solution, one solution or infinitely many.
- Recognise an **identity** 恒等式, an equation true for every value.
- Interpret the **slope** 斜率 and **intercept** 截距 of a model with units.
- Classify a system of two linear equations, and solve an **inequality** 不等式 in one or two variables.

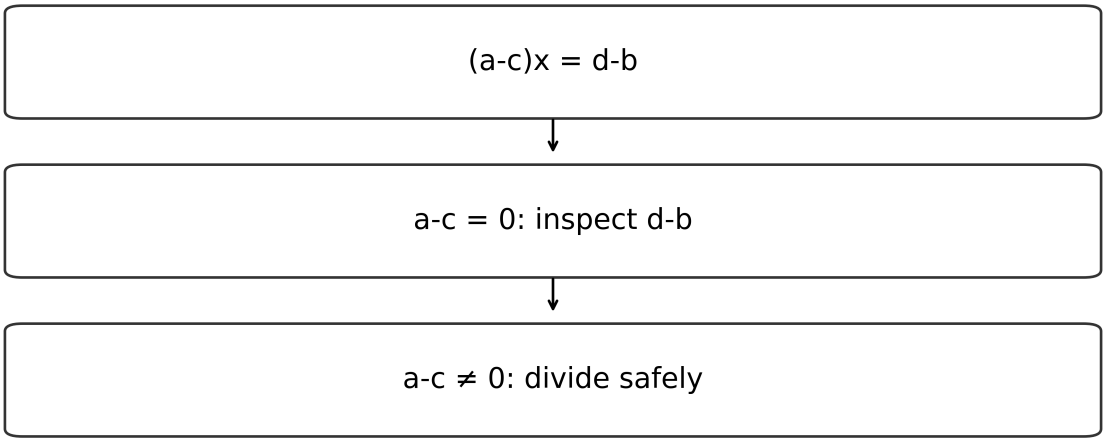
Counting the solutions of one equation

Collect the variable terms on one side and the constants on the other:

$$ax + b = cx + d \;\Rightarrow\; (a - c)x = d - b$$

- If $a - c \neq 0$, divide: one solution, $x = \dfrac{d - b}{a - c}$.
- If $a - c = 0$ and $d - b = 0$, the equation reads $0 = 0$: an identity, with infinitely many solutions.
- If $a - c = 0$ but $d - b \neq 0$, it reads $0 =$ (a non-zero number): no solution.
- Never divide by an expression containing the parameter until you know it is not zero.

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Worked steps: (a-c)x = d-b → a-c = 0: inspect d-b → a-c ≠ 0: divide safely

Linear models with units

- In $y = mx + b$, the slope m is the change in y per one unit of x, so its units are "units of y per unit of x".
- The intercept b is y when $x = 0$: often a fixed fee or a starting amount.
- An intercept can exist algebraically but mean nothing if $x = 0$ is outside the situation.
- Given two data points, find the slope first, then substitute one point to find b.

Systems of two linear equations

Compare the two equations before solving:

- If one equation is a multiple of the other (both sides), they are the same line: infinitely many solutions.
- If the x and y coefficients are in the same ratio but the constants are not, the lines are parallel: no solution.
- Otherwise the lines cross once. Solve by elimination or substitution, and check the pair in both original equations.

Inequalities

- Solve as for an equation, except that multiplying or dividing by a negative number reverses the sign: $-2x < 6$ gives $x > -3$.
- A strict inequality ($<$, $>$) excludes the boundary; $\leq$ and $\geq$ include it.
- In two variables, graph the boundary line (dashed if strict, solid if not), then test a point such as $(0, 0)$ to choose the side.

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- In a word problem, the solution must also be sensible: whole tickets, non-negative hours.

Worked example: a system with a parameter

For which k and c does the system $2x + 3y = 7$, $4x + ky = c$ have infinitely many solutions, no solution, or one?

- Doubling the first equation gives $4x + 6y = 14$.
- If $k = 6$ and $c = 14$, the equations are the same line: infinitely many solutions.
- If $k = 6$ but $c \neq 14$, the lines are parallel: no solution.
- If $k \neq 6$, the slopes differ: exactly one solution, whatever c is.

Worked example: a budget inequality

Adult tickets cost 8 yuan and child tickets 5 yuan. A group has 100 yuan.

- With a adults and c children: $8a + 5c \leq 100$, with a, c whole numbers at least 0.
- Check $(10, 4)$: $80 + 20 = 100 \leq 100$, so it is allowed; the boundary is included.
- Check $(10, 5)$: $80 + 25 = 105 > 100$, not allowed.
- With 6 adults, $5c \leq 52$, so $c \leq 10.4$: at most 10 children.

Common errors

- Dividing by $k - 3$ without first checking the case $k = 3$.
- Forgetting to reverse the inequality sign when dividing by a negative.
- Calling a system "no solution" when the constants also match (it is the same line).
- Giving the intercept as a meaning when $x = 0$ is impossible in the context.
- Shading a half-plane from the slope instead of testing a point.

Key terms

parameter: A letter that stands for a fixed but unknown number in an equation.

identity: An equation that is true for every allowed value of the variable.

slope: The change in y for each one-unit increase in x.

intercept: The value of y when $x = 0$.

inequality: A statement that one quantity is greater or less than another.

Check yourself

- For which k does $kx - 4 = 3x + 2$ have no solution?
- Solve $5 - 2x > 11$.
- Does $(1, 2)$ satisfy $y < 3x - 1$?

Answers: $k = 3$, since it becomes $0x = 6$. $-2x > 6$, so $x < -3$. No: $2 < 2$ is false.

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Equivalent expressions and domain restrictions

SAT

Around 820 the scholar Muhammad ibn Musa al-Khwarizmi, working in Baghdad's House of Wisdom, wrote a book on solving equations. Its title named two moves: *al-jabr*, "restoring" a subtracted term to the other side, and *al-muqabala*, "balancing" equal terms on both sides. Latin translators turned *al-jabr* into "algebra". Every rewrite in algebra must keep an expression's value the same; the SAT tests whether you can do that, and whether you notice the values where a rewrite stops being true.

These questions ask you to:

- Recognise **equivalent expressions** 等价表达式, equal for every allowed value.
- Expand brackets and **factor** 因式 expressions using standard patterns.
- State the **domain** 定义域 restrictions of a rational expression.
- Rewrite roots and powers correctly.

What "equivalent" means

- Two expressions are equivalent if they give the same value for every input in their domain.
- One matching test value does not prove equivalence: x^2 and $2x$ agree at $x = 2$ but not at $x = 3$.
- One different value does prove they are not equivalent.
- If two polynomials are equal for all x, their matching coefficients are equal. This lets you find unknown constants.

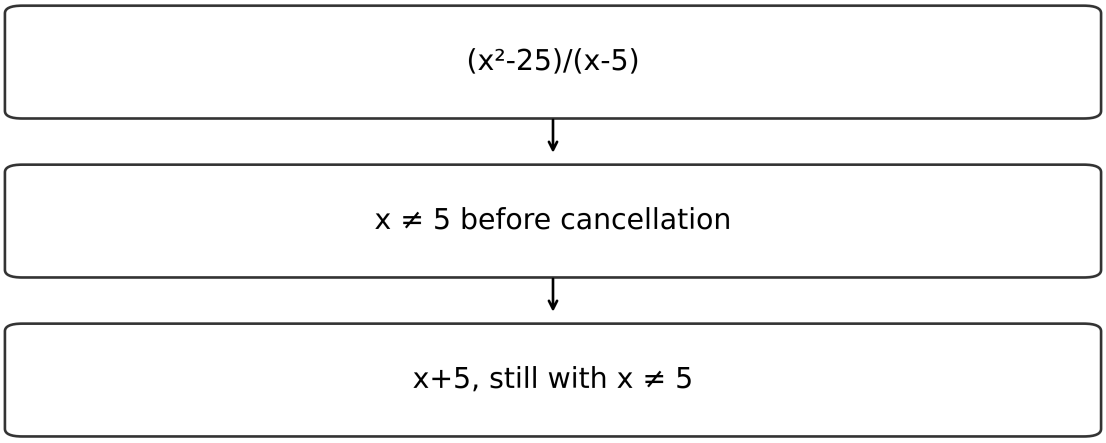
Expanding and factoring

$$(a+b)^2 = a^2 + 2ab + b^2 \qquad a^2 - b^2 = (a-b)(a+b)$$

- Expand every term in one bracket by every term in the other.
- Factor by first taking out a common factor, then looking for a difference of squares or a trinomial.
- For $x^2 + bx + c$, find two numbers with product c and sum b.

Rational expressions and restrictions

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Worked steps: $(x^2$-25)/(x-5) → $x \neq 5$ before cancellation → x+5, still with $x \neq 5$

- Find the restrictions from the original denominator before you cancel anything.
- Cancel only common factors, never common terms: $\dfrac{x+3}{x}$ is not 3.
- After cancelling, the restriction stays: $\dfrac{x^2-25}{x-5} = x+5$ only for $x \neq 5$.

Roots and powers

- $x^{1/n} = \sqrt[n]{x}$ and $x^{m/n} = (\sqrt[n]{x})^m$.
- $\sqrt{x^2} = |x|$, not x: at $x = -3$ it gives 3.
- $x^a \cdot x^b = x^{a+b}$ and $(x^a)^b = x^{ab}$, but $(x+y)^2 \neq x^2 + y^2$.

Worked example: a rational expression

Simplify $F(x) = \dfrac{x^2-4x}{x^2-16}$ and state its restrictions.

- Factor: numerator $x(x-4)$, denominator $(x-4)(x+4)$.
- The original denominator is zero at $x = 4$ and $x = -4$, so both are excluded.
- Cancel: $F(x) = \dfrac{x}{x+4}$ for $x \neq 4, -4$.
- $\dfrac{x}{x+4}$ alone allows $x = 4$ (it gives $\frac{1}{2}$), but $F(4)$ is undefined, so they are not identical.

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Worked example: matching coefficients

If $(ax+3)(x-2) = 4x^2 + bx - 6$ for all x, find a and b.

- Expand: $ax^2 - 2ax + 3x - 6 = ax^2 + (3-2a)x - 6$.
- Match x^2: $a = 4$.
- Match x: $b = 3 - 8 = -5$.
- Check at $x = 1$: left $(7)(-1) = -7$; right $4 - 5 - 6 = -7$.

Common errors

- Cancelling terms across a fraction bar instead of factors.
- Dropping a restriction after cancelling.
- Writing $(x+3)^2 = x^2 + 9$.
- Writing $\sqrt{x^2} = x$ for negative x.
- Treating one matching test value as proof.

Key terms

equivalent expressions: Expressions with the same value for every allowed input.

factor: An expression that multiplies with another to give a product; to factor is to write as such a product.

domain: The set of input values for which an expression is defined.

Check yourself

- Expand $(x-3)(x+3)$.
- State the restrictions on $\dfrac{x+1}{x^2-9}$.
- Is $\sqrt{x^2} = x$ for every real x?

Answers: $x^2 - 9$. $x \neq 3$ and $x \neq -3$. No: it equals $|x|$, so it fails for $x = -2$.

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Nonlinear equations and mixed systems

SAT

In 1545 the Italian doctor and mathematician Gerolamo Cardano published *Ars Magna* ("The Great Art"), with the first printed method for solving cubic equations. He had learnt the key step from Niccolò Tartaglia, who had made him swear to keep it secret, and the two men quarrelled for years. Cardano's formulas sometimes produced square roots of negative numbers, even when the answer was an ordinary real number. Nonlinear equations can give answers that look right but are not; this handout shows how to find every solution and reject the false ones.

These questions ask you to:

- Solve quadratic equations by factoring, completing the square or the formula.
- Use the **discriminant** 判别式 to count real solutions.
- Check each **candidate root** 候选根 from a radical, rational or absolute-value equation.
- Find the **intersection** 交点 points of a line and a **parabola** 抛物线.

Solving quadratics

$$ax^2 + bx + c = 0 \;\Rightarrow\; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Factor when the numbers are small; use the formula otherwise.
- Put everything on one side first. Do not divide by x: in $x^2 = 9x$ that loses the solution $x = 0$.
- Completing the square, $x^2 + 6x = (x + 3)^2 - 9$, also gives the vertex.

The discriminant

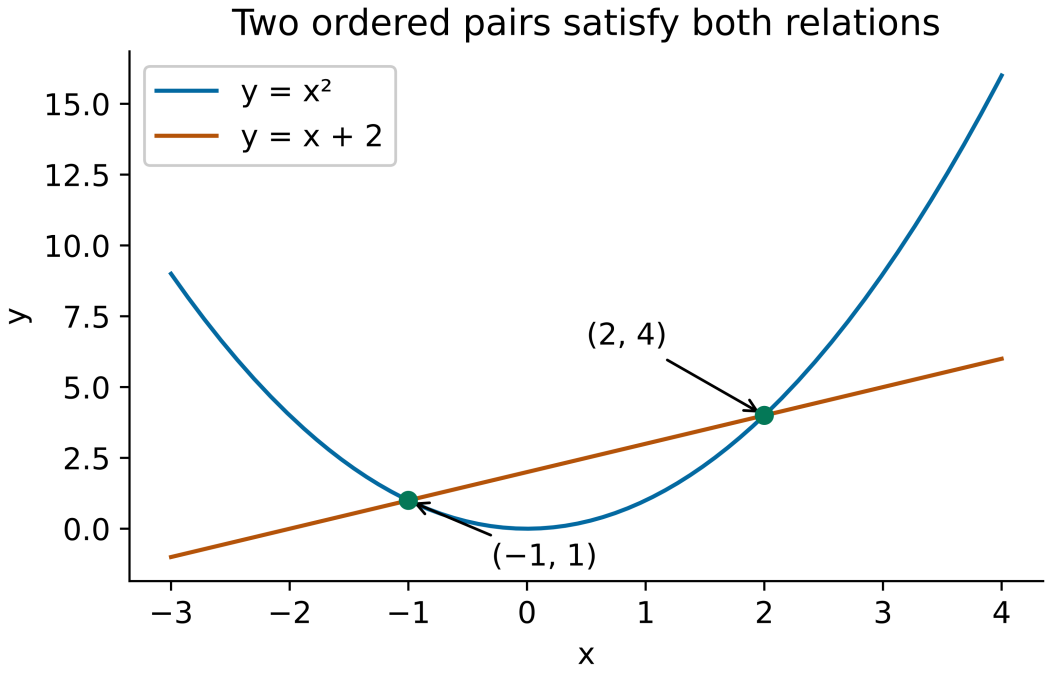
- $b^2 - 4ac > 0$: two real solutions; the graph crosses the x-axis twice.
- $b^2 - 4ac = 0$: one repeated solution; the graph touches the axis.
- $b^2 - 4ac < 0$: no real solutions.

Radical, rational and absolute-value equations

- Radical: isolate the root, square both sides, solve, then check every candidate in the original. Squaring can create a false solution.
- Rational: list the excluded values first, then multiply through by the denominator; reject any candidate that is excluded.
- Absolute value: $|A| = k$ (with $k \geq 0$) splits into $A = k$ or $A = -k$.

Mixed systems

- To solve a line with a parabola, set the two expressions for y equal, giving a quadratic.
- Its discriminant tells you whether the line crosses, touches or misses the parabola.
- Find each y from the simpler (linear) equation.



For the figure: $x^2 = x + 2$ gives $x^2 - x - 2 = (x - 2)(x + 1) = 0$, so $x = 2$ or $x = -1$, and the points are $(2, 4)$ and $(-1, 1)$.

Worked example: a radical equation

Solve $\sqrt{2x + 3} = x$.

- A square root is never negative, so $x \geq 0$.
- Square: $2x + 3 = x^2$, so $x^2 - 2x - 3 = (x - 3)(x + 1) = 0$.
- Candidates: $x = 3$ and $x = -1$.
- Check: $\sqrt{9} = 3$ works; $\sqrt{1} = 1 \neq -1$ fails. Only $x = 3$.

Worked example: a line touching a parabola

For which c does the line $y = x + c$ meet $y = x^2$ at exactly one point?

- Set equal: $x^2 = x + c$, so $x^2 - x - c = 0$.
- One point means discriminant zero: $(-1)^2 - 4(1)(-c) = 1 + 4c = 0$.
- So $c = -\frac{1}{4}$, and the touching point has $x = \frac{1}{2}$, $y = \frac{1}{4}$.

Common errors

- Dividing both sides by x and losing $x = 0$.
- Keeping a candidate from squaring without checking it.
- Forgetting the negative case of an absolute value.
- Keeping a solution that makes a denominator zero.
- Writing only the x-values when the question asks for points.

Key terms

discriminant: The value $b^2 - 4ac$, which tells how many real solutions a quadratic has.

candidate root: A possible solution that must still be checked in the original equation.

intersection: A point that lies on both graphs.

parabola: The U-shaped graph of a quadratic function.

Check yourself

- How many real solutions has $x^2 + 4x + 5 = 0$?
- Solve $|2x - 1| = 5$.
- Solve $x^2 = 9x$.

Answers: none, since $16 - 20 = -4$. $x = 3$ or $x = -2$. $x = 0$ or $x = 9$.

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Nonlinear functions, transformations and growth

SAT

Nonlinear Functions: Transformations and Growth

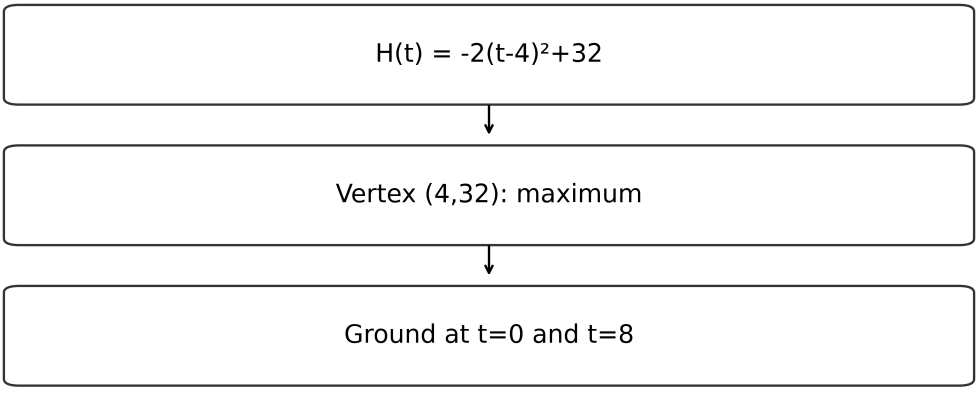
In 1798 the English clergyman Thomas Malthus published *An Essay on the Principle of Population*. He argued that a population, unchecked, grows geometrically (1, 2, 4, 8, …) while food supply grows only arithmetically (1, 2, 3, 4, …), so hunger must follow. He was wrong about food, which grew much faster than he expected, but his contrast between adding a fixed amount and multiplying by a fixed factor is exactly the difference between linear and exponential functions.

These questions ask you to:

- Read a quadratic in standard, factored and **vertex form** 顶点式, and find the vertex of a **parabola** 抛物线.
- Describe a **transformation** 变换 of a graph from its equation.
- Write and interpret **exponential growth** 指数增长 and decay models.
- Tell linear from exponential change in a table.

Three forms of a quadratic

- Standard, $y = ax^2 + bx + c$: the y-intercept is c.
- Factored, $y = a(x - p)(x - q)$: the x-intercepts are p and q.
- Vertex, $y = a(x - h)^2 + k$: the vertex is (h, k). If $a < 0$, k is the maximum; if $a > 0$, the minimum.
- The vertex lies halfway between the x-intercepts.



Worked steps: H(t) = -2(t-4)²+32 → Vertex (4,32): maximum → Ground at t=0 and t=8

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Transformations

- $f(x) + k$ moves the graph up k; $f(x - h)$ moves it right h (note the minus sign).
- $af(x)$ stretches it vertically by a; a negative a also reflects it in the x-axis.
- $f(-x)$ reflects it in the y-axis.
- Track one point, such as the vertex, through each step.

Exponential models

$$N(t) = N_0\, b^{t/T}$$

- N_0 is the starting amount; b is the factor for each period of length T.
- Growth of r per cent per period means $b = 1 + r/100$; decay means $b = 1 - r/100$.
- A factor of 1.5 is 50% growth, not 150%.
- Over two periods the factor is b^2, not $2b$.

Linear or exponential?

- Equal steps in x give equal differences in y: linear.
- Equal steps in x give equal ratios in y: exponential.
- Exponential growth always overtakes linear growth eventually.

Worked example: a projectile

A model height is $H(t) = -3(t-2)^2 + 12$ metres, t in seconds, from launch until it lands.

- Ground: $H = 0$ gives $(t-2)^2 = 4$, so $t = 0$ or $t = 4$. Domain $0 \le t \le 4$.
- $a = -3 < 0$, so the vertex $(2, 12)$ is a maximum: 12 m at 2 s.
- At least 9 m: $(t-2)^2 \le 1$, so $1 \le t \le 3$ seconds.

Worked example: growth with a period

A culture starts with 120 cells and grows by a factor of 1.5 every two hours.

- $N(t) = 120(1.5)^{t/2}$, with t in hours.
- $N(4) = 120(1.5)^2 = 120 \times 2.25 = 270$ cells.
- The growth is 50% per two hours. The hourly factor is $\sqrt{1.5} \approx 1.22$, about 22% per hour, not 25%.

Common errors

- Reading the vertex of $y = (x+3)^2$ as $(3, 0)$ instead of $(-3, 0)$.
- Writing a 20% decay factor as 0.2 instead of 0.8.
- Using t instead of $t/2$ when the factor is per two hours.

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- Calling a table linear because its values increase.
- Forgetting the domain of a real-world model.

Key terms

vertex form: $y = a(x - h)^2 + k$, which shows the vertex (h, k).
parabola: The U-shaped graph of a quadratic function.
transformation: A change to a graph, such as a shift, stretch or reflection.
exponential growth: Growth by the same factor in each equal period.

Check yourself

- Find the vertex of $y = 2(x + 3)^2 - 7$. Is it a maximum or a minimum?
- A car's value falls 20% a year. What is the yearly factor?
- The table gives 5, 10, 20, 40 for $x = 0, 1, 2, 3$. Linear or exponential?

Answers: $(-3, -7)$, a minimum. 0.8. Exponential, with factor 2.

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Ratios, unit conversion and successive percentages

SAT

On 23 July 1983 an Air Canada Boeing 767 ran out of fuel at about 41,000 feet over Manitoba. Canada had recently moved to metric units. The crew needed 22,300 kilograms of fuel for the trip, but a conversion factor for pounds was used where one for kilograms was needed, and the aircraft carried less than half that. The pilots glided the plane to a disused air base at Gimli, and everyone survived. The SAT does not set life-or-death problems, but it does test the same skill: carry units through every step.

These questions ask you to:

- Share amounts in a **ratio** 比.
- Find and compare a **unit rate** 单位率.
- Convert units with conversion factors, including area and volume.
- Work with a **percentage change** 百分比变化, including several changes in a row.

Ratios

- In a ratio $a : b$, the whole has $a + b$ parts. One part is the total divided by $a + b$.
- Ratios can be scaled: $4 : 6 = 2 : 3$.
- A part-to-part ratio is not a fraction of the whole: in $2 : 3$, the first is $\frac{2}{5}$ of the total, not $\frac{2}{3}$.

Rates and unit rates

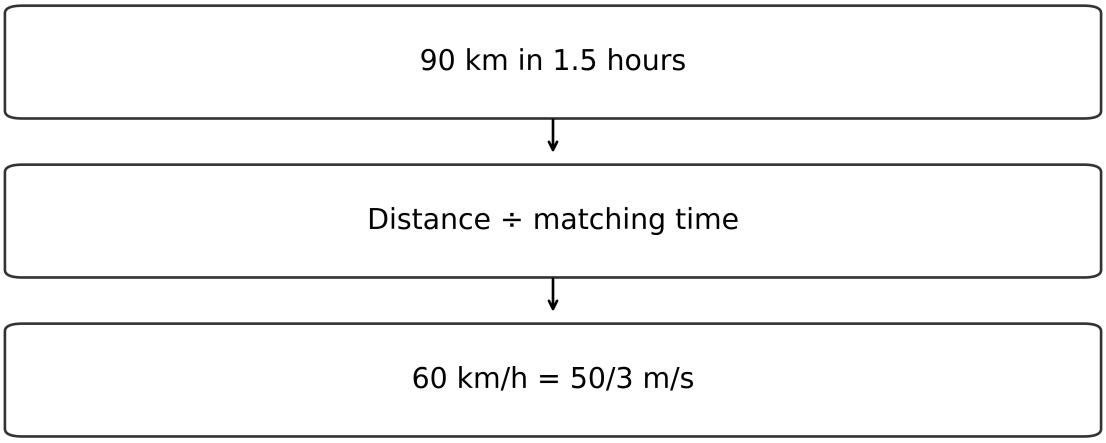
- A rate compares quantities with different units: kilometres per hour, yuan per kilogram.
- A unit rate gives the amount for one unit. Divide to find it, then compare like with like.
- Time = amount ÷ rate.

Unit conversion

$$60\ \frac{\text{km}}{\text{h}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{h}}{3600\ \text{s}} = \frac{50}{3}\ \frac{\text{m}}{\text{s}}$$

- Write each factor as a fraction equal to 1, arranged so the unwanted units cancel.
- For area, square the factor: $1\ \text{m}^2 = 10\,000\ \text{cm}^2$. For volume, cube it.
- Or convert lengths first, then square or cube.

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Worked steps: 90 km in 1.5 hours → Distance ÷ matching time → 60 km/h = 50/3 m/s

Percentages

- Percentage change $= \dfrac{\text{new} - \text{original}}{\text{original}} \times 100\%$.
- An increase of $r\%$ multiplies by $1 + r/100$; a decrease multiplies by $1 - r/100$.
- Successive changes multiply: they do not add.
- Reverse a change by dividing: if 96 is after a 20% rise, the original is $96 \div 1.2 = 80$.

Worked example: a ratio with costs

Red and blue paint are mixed in the ratio $4 : 7$ to make 33 litres. Red costs 18 yuan per litre and blue 12 yuan per litre.

- Parts: $4 + 7 = 11$, so one part is $33 \div 11 = 3$ L.
- Red $= 12$ L and blue $= 21$ L.
- Cost $= 12 \times 18 + 21 \times 12 = 216 + 252 = 468$ yuan.

Worked example: two percentage changes

A 400-yuan item falls 15% and then rises 10%.

- Final price $= 400 \times 0.85 \times 1.10 = 374$ yuan.
- Overall change $= \dfrac{374 - 400}{400} = -0.065$: a 6.5% decrease, not 5%.
- To return to 400 yuan from 374 would need a rise of $26 \div 374 \approx 7.0\%$.

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Common errors

- Adding successive percentages instead of multiplying factors.
- Dividing a reverse percentage by the wrong base.
- Using 100 instead of 10,000 for square metres to square centimetres.
- Treating a part-to-part ratio as a fraction of the whole.
- Dropping units halfway through a calculation.

Key terms

ratio: A comparison of two quantities by division, written $a : b$.

unit rate: A rate given for one unit of the second quantity.

percentage change: The change divided by the original value, as a percentage.

Check yourself

- Convert 72 km/h to m/s.
- A price falls 30% and then rises 30%. What is the overall change?
- How many cm^2 are in 2.5 m^2?

Answers: 20 m/s. $0.7 \times 1.3 = 0.91$, a 9% decrease. 25,000 cm^2.

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Distributions, transformations and spread

SAT

In 1906, at a livestock fair in Plymouth, England, about 800 visitors paid to guess the weight of an ox after butchering. The scientist Francis Galton later studied the 787 readable tickets. In *Nature* in 1907 he reported that the middle guess, the median, was 1,207 pounds; the true weight was 1,198 pounds, an error of under 1%. Galton chose the median because a few wild guesses could not pull it far. Choosing the right measure of centre, and of spread, is what SAT data questions test.

These questions ask you to:

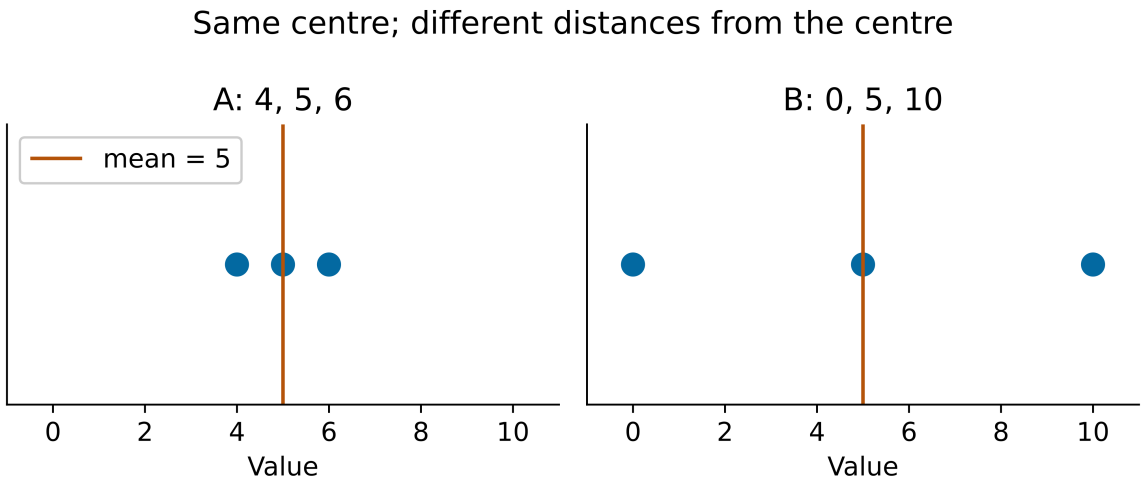
- Find the mean and the **median** 中位数, and choose between them.
- Read a **frequency** 频数 table, dot plot or histogram.
- Compare spread using range, the **interquartile range** 四分位距 and the **standard deviation** 标准差.
- Predict how an **outlier** 离群值 or a change to every value affects each measure.

Centre: mean and median

- The mean is the sum divided by the number of values; the median is the middle value in order (the mean of the middle two if there is an even number).
- An outlier pulls the mean towards it but hardly moves the median.
- In a right-skewed distribution (a long tail of high values), the mean is greater than the median.

Spread

- Range = largest − smallest. It depends only on the two extremes.
- IQR = upper quartile − lower quartile: the spread of the middle half.
- Standard deviation measures the typical distance of values from the mean. The SAT does not ask you to calculate it, only to compare.
- Data bunched near the mean have a smaller standard deviation.



Both sets in the figure have mean 5, but B's values lie farther from it, so B has the larger standard deviation and range.

Frequency tables and graphs

- In a frequency table, each value counts as many times as its frequency.
- Total n is the sum of the frequencies; the median is at position $(n+1)/2$.
- On a histogram, read the bar heights as counts for each interval, not single values.

Changing every value

- Adding k to every value adds k to the mean and median; the range, IQR and standard deviation do not change.
- Multiplying every value by k (positive) multiplies every measure of centre and spread by k.
- For $z = 3x - 4$: centre $\times 3$ then -4; spread $\times 3$ only.

Worked example: a frequency table

Values 2, 5 and 8 have frequencies 3, 2 and 1.

- The data are 2, 2, 2, 5, 5, 8, so $n = 6$.
- Mean $= \dfrac{3(2) + 2(5) + 1(8)}{6} = \dfrac{24}{6} = 4$.
- Median = mean of the 3rd and 4th values $= \dfrac{2+5}{2} = 3.5$. Range $= 8 - 2 = 6$.

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Worked example: transforming the data

Every value above is changed by $z = 3x - 4$.

- New mean $= 3(4) - 4 = 8$; new median $= 3(3.5) - 4 = 6.5$.
- New range $= 3 \times 6 = 18$; the "-4" moves every value equally and does not change the spread.
- The standard deviation also triples.

Common errors

- Forgetting to put the data in order before finding the median.
- Using the number of rows instead of the total frequency as n.
- Saying that adding a constant changes the standard deviation.
- Comparing spread with range alone when one value is an outlier.
- Choosing the mean for strongly skewed data, such as incomes.

Key terms

median: The middle value of ordered data.

frequency: The number of times a value occurs.

interquartile range: The upper quartile minus the lower quartile.

standard deviation: A measure of how far values typically lie from the mean.

outlier: A value far from the rest of the data.

Check yourself

- For 3, 4, 4, 5, 24, find the mean and median. Which better describes a typical value?
- Adding 10 to every value: what happens to the standard deviation?
- Which has the larger standard deviation: {4, 5, 6} or {0, 5, 10}?

Answers: mean 8, median 4; the median, because 24 is an outlier. It does not change. {0, 5, 10}.

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Scatterplots, model residuals and predictions

SAT

On 1 January 1801 the astronomer Giuseppe Piazzi found a faint moving object, later named Ceres, and tracked it for 41 days before it disappeared behind the Sun. Astronomers could not tell where it would reappear. The 24-year-old Carl Friedrich Gauss fitted an orbit to Piazzi's few observations, choosing the curve that made the squared errors as small as possible. In December 1801 Ceres was found almost exactly where he predicted. Adrien-Marie Legendre published this "least squares" method in 1805. The SAT uses its simplest case: a line of best fit through a scatterplot.

These questions ask you to:

- Describe the pattern in a scatterplot and read a **line of best fit** 最佳拟合直线.
- Interpret the slope and intercept of a fitted model in context.
- Find and interpret a **residual** 残差.
- Tell **interpolation** 内插 from **extrapolation** 外推, and judge when a prediction can be trusted.

Reading a scatterplot

- Direction: positive (up to the right) or negative.
- Form: linear, curved (such as exponential), or no clear pattern.
- Strength: how closely the points cluster around the pattern.
- An association between two variables does not show that one causes the other.

The fitted line

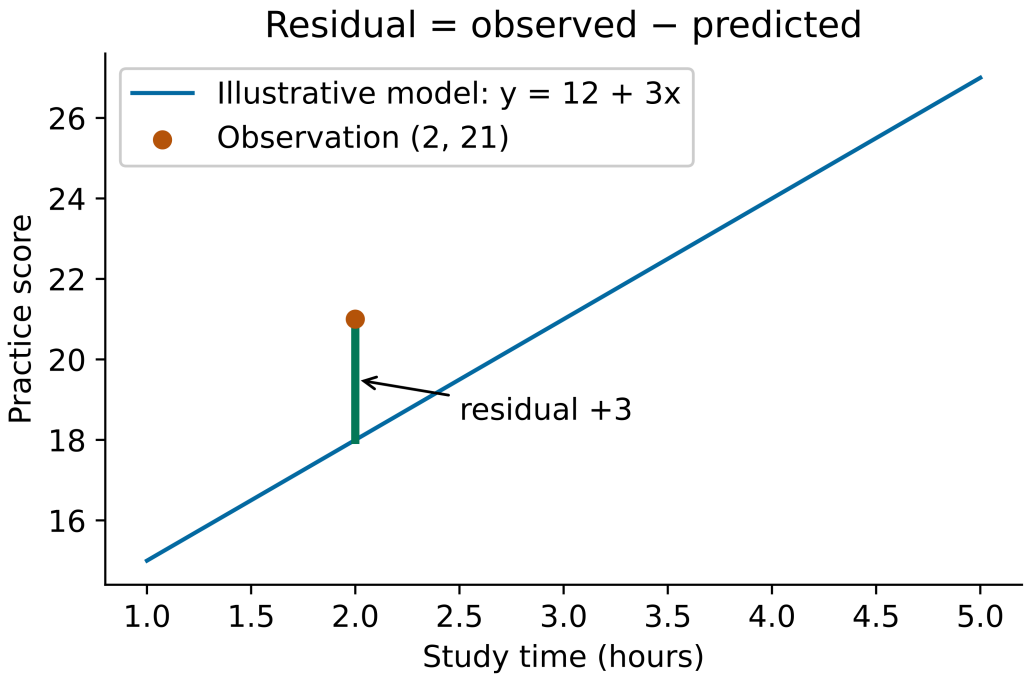
- In $\hat{y} = a + bx$, the slope b is the predicted change in y for each one-unit increase in x. Say "predicted", since real points vary.
- The intercept a is the predicted y at $x = 0$. If $x = 0$ is outside the data, it is only an extrapolation.
- To predict, substitute x into the equation, or read from the line, not from the nearest point.

Residuals

$$\text{residual} = \text{observed } y - \text{predicted } y$$

- A positive residual means the point lies above the line; negative means below.
- On a residual plot, a random scatter around zero suggests the linear model fits; a curve suggests another form.

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In the figure, the line predicts $12 + 3(2) = 18$ at $x = 2$, so the residual is $21 - 18 = 3$.

Linear or exponential models

- If equal steps in x give roughly equal differences in y, a linear model fits.
- If they give roughly equal ratios, an exponential model fits: $y = ab^x$.
- Look at the shape of the scatterplot before choosing.

Worked example: a delivery model

A fitted model is $T = 8 + 1.5d$ minutes, for observed distances of 2 to 10 km.

- Slope: each extra kilometre adds a predicted 1.5 minutes.
- Intercept: 8 minutes at 0 km, but 0 km is outside the data, so this is an extrapolation (it might be loading time).
- At 6 km: $T = 8 + 1.5(6) = 17$ minutes. The observed time was 15, so the residual is $15 - 17 = -2$ minutes: faster than predicted.

Worked example: choosing a model

At $x = 0, 1, 2, 3$ the outputs are 6, 18, 54, 162.

- Differences: 12, 36, 108, not constant. Ratios: all 3.
- Model: $y = 6(3)^x$.
- Prediction at $x = 4$: $6 \times 81 = 486$, but only if the growth continues, since $x = 4$ is outside the data.

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Common errors

- Getting the residual the wrong way round (predicted minus observed).
- Saying the slope "causes" the change.
- Trusting a prediction far outside the observed range.
- Reading a prediction from a nearby point instead of from the line.
- Fitting a line to clearly curved data.

Key terms

line of best fit: A line that models the trend in a scatterplot.

residual: The observed value minus the value the model predicts.

interpolation: Predicting inside the range of the data.

extrapolation: Predicting outside the range of the data.

Check yourself

- A model is $\hat{y} = 40 - 2x$. At $x = 5$ the observed value is 33. Find the residual.
- The data run from $x = 1$ to $x = 12$. Is a prediction at $x = 30$ interpolation or extrapolation?
- What does a negative residual tell you about a point?

Answers: predicted 30, residual +3. Extrapolation. It lies below the line.

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Probability from a table and a restricted group

SAT

In 1654 a French gambler, the Chevalier de Méré, asked Blaise Pascal a question: if a game of chance is stopped early, how should the stakes be shared fairly? Pascal wrote to Pierre de Fermat in Toulouse, and in a short exchange of letters that summer the two worked out the answer by counting every way the game could have finished. Their letters are often called the start of probability theory. SAT probability questions are counting questions too, usually from a table, and the hard part is choosing the right group to count within.

These questions ask you to:

- Find probabilities from a **two-way table** 双向表.
- Find a **conditional probability** 条件概率, the probability within a restricted group.
- Decide whether two events are **independent events** 独立事件.
- Use “and”, “or” and “not” correctly.

Probability from counts

$$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}}$$

- Probabilities lie between 0 and 1. $P(\text{not } A) = 1 - P(A)$.
- In a table, the total is usually the grand total, unless the question restricts the group.

Restricted groups

- “Given that”, “of those who”, “among” all restrict the group: the denominator becomes that row or column total.
- $P(A \mid B) = \dfrac{\text{number in both } A \text{ and } B}{\text{number in } B}$.
- $P(A \mid B)$ and $P(B \mid A)$ are usually different, because they have different denominators.

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Original counts: condition on the requested group

Group	Lunch	No lunch	Total
Bus	18	12	30
Not bus	12	8	20
Total	30	20	50

Independence

- Events A and B are independent if knowing B does not change the probability of A: $P(A \mid B) = P(A)$.
- Independent is not the same as disjoint. Disjoint events cannot happen together; if both have positive probability, they are not independent.

"Or" and "and"

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, so the overlap is not counted twice.
- In a table, it is often easier to count the cells directly.
- Expected count = probability $\times$ number of trials.

Worked example: the lunch table

Use the table in the figure.

- $P(\text{lunch}) = \dfrac{18 + 12}{50} = \dfrac{30}{50} = 0.6$.
- $P(\text{lunch} \mid \text{bus}) = \dfrac{18}{30} = 0.6$.
- These are equal, so in this table bringing lunch is independent of using the bus.
- $P(\text{bus} \mid \text{lunch}) = \dfrac{18}{30} = 0.6$, and $P(\text{bus}) = \dfrac{30}{50} = 0.6$ as well.

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Worked example: a table that is not independent

Of 20 cyclists, 12 drink tea; of 30 walkers, 9 drink tea.

- $P(\text{tea}) = \dfrac{21}{50}$; $P(\text{tea} \mid \text{cyclist}) = \dfrac{12}{20} = \dfrac{3}{5}$. These differ, so the events are not independent.
- $P(\text{cyclist} \mid \text{tea}) = \dfrac{12}{21} = \dfrac{4}{7}$: a different denominator, so a different answer.
- $P(\text{walker or tea}) = \dfrac{30 + 21 - 9}{50} = \dfrac{42}{50}$; the 9 tea-drinking walkers are counted once.

Common errors

- Using the grand total when the question restricts the group.
- Mixing up $P(A \mid B)$ and $P(B \mid A)$.
- Counting the overlap twice in an "or" question.
- Confusing independent with disjoint.
- Reading the wrong row or column of the table.

Key terms

two-way table: A table that counts outcomes by two categories at once.

conditional probability: The probability of an event within a restricted group.

independent events: Events where one happening does not change the probability of the other.

Check yourself

- Of 16 boys, 10 play chess. What is $P(\text{chess} \mid \text{boy})$?
- $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \text{ and } B) = 0.1$. Find $P(A \text{ or } B)$.
- If $P(A \mid B) = P(A)$, what can you say about A and B?

Answers: $\frac{10}{16} = \frac{5}{8}$. $0.5 + 0.3 - 0.1 = 0.7$. They are independent.

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Sampling, margin of error and causal claims

SAT

In 1936 the magazine *Literary Digest* mailed about 10 million ballots asking Americans how they would vote for president. About 2.4 million came back, and the magazine predicted that Alf Landon would beat Franklin Roosevelt. Roosevelt won 46 of the 48 states. The *Digest*'s lists came largely from telephone directories and car registrations, and its answers came only from people who chose to reply, so its huge sample was biased. George Gallup, polling only about 50,000 people chosen more carefully, called the winner correctly. How a sample is chosen matters more than how big it is.

These questions ask you to:

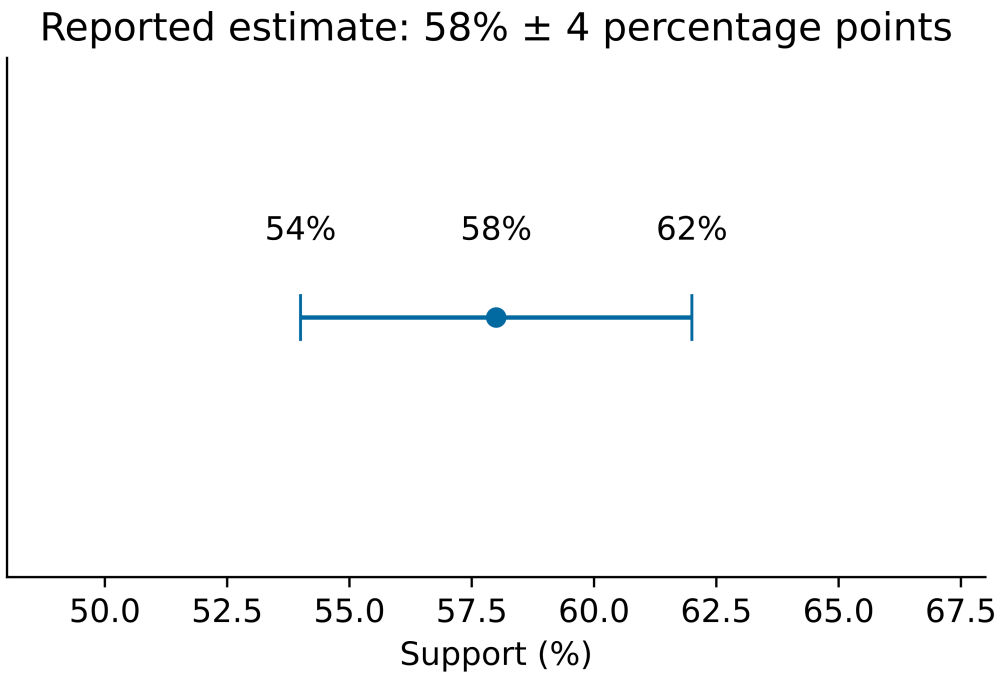
- Decide whether a sample's result can be applied to a population, using whether it was a **random sample** 随机样本.
- Interpret an estimate and its **margin of error** 误差范围.
- Recognise **sampling bias** 抽样偏差.
- Decide whether a study supports a causal claim, using whether it used **random assignment** 随机分组.

Samples and populations

- The population is the whole group the question is about; the sample is the part actually measured.
- Results from a random sample can be generalised to the population it was drawn from, and only to that population.
- A sample of volunteers, or of people easy to reach, may differ from the population in a systematic way.

Margin of error

- An estimate of 58% with a margin of error of 4 percentage points gives the plausible interval 54% to 62%.
- A larger random sample gives a smaller margin of error: roughly, four times the sample size halves it.
- The margin covers random sampling variation only. It does not correct bias.



Observational studies and experiments

- An observational study records what people already do. It can show an association, but other factors may explain it.
- An experiment assigns treatments. With random assignment, groups are alike on average, so a difference can be put down to the treatment.
- Random sampling allows generalisation; random assignment allows cause. A study may have one, both or neither.

Judging claims

- Check three things: the population, the method of selection, and whether treatments were randomly assigned.
- The right answer usually limits the claim to the group studied.
- Watch for “proves”, “all” and “every”: they are usually too strong.

Worked example: a library survey

A random sample of 500 registered library members finds 62% support for longer opening hours, with a margin of error of 4 percentage points.

- Plausible interval: 58% to 66% of registered members.
- The population is registered members, so the result says nothing reliable about town residents who are not members.
- It is plausible that a majority of members support the change, since the whole interval is above 50%.

Worked example: an experiment with volunteers

Forty volunteers are randomly assigned to old or new instructions for the same task. The new group finishes faster on average.

- Random assignment supports the causal claim that the new instructions made these participants faster.
- The participants were volunteers, not a random sample, so the result may not apply to all users.
- A reasonable conclusion: "For people like these volunteers, the new instructions probably cause faster completion."

Common errors

- Generalising beyond the population that was sampled.
- Believing a very large sample cannot be biased.
- Claiming cause from an observational study.
- Treating the margin of error as covering all possible errors.
- Confusing random sampling with random assignment.

Key terms

random sample: A sample in which every member of the population has a known, fair chance of being chosen.

margin of error: The amount added to and subtracted from an estimate to allow for random sampling variation.

sampling bias: A systematic difference between the sample and the population caused by how the sample was chosen.

random assignment: Using chance to decide which treatment each participant receives.

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Check yourself

- An estimate is 45% with a margin of error of 3 points. What is the interval?
- A town surveys gym members about how often residents exercise. What is the problem?
- An observational study finds coffee drinkers live longer. Does coffee cause longer life?

Answers: 42% to 48%. Sampling bias: gym members exercise more than typical residents. Not shown: other factors may explain the association.

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Scale factors, angle relationships and triangles

SAT

Around 240 BC Eratosthenes, the librarian at Alexandria, heard that at noon on midsummer's day the Sun shone straight down a well at Syene, to the south. At the same moment in Alexandria a vertical stick cast a shadow at an angle of about 7.2°, one fiftieth of a full circle. Because the Sun's rays arrive parallel, the angle between the two places at Earth's centre must also be 7.2°. Syene was about 5,000 stadia away, so Earth's circumference was about $50 \times 5{,}000 = 250{,}000$ stadia, close to the true value. He used parallel lines, equal angles and proportion: the core of SAT geometry.

These questions ask you to:

- Use angle facts for lines, parallel lines and triangles.
- Use **similarity** 相似 and the **scale factor** 比例因子 for lengths, areas and volumes.
- Recognise **congruence** 全等 conditions.
- Use Pythagoras's theorem, special right triangles and sine, cosine and tangent, and find areas with the **perpendicular height** 垂直高度.

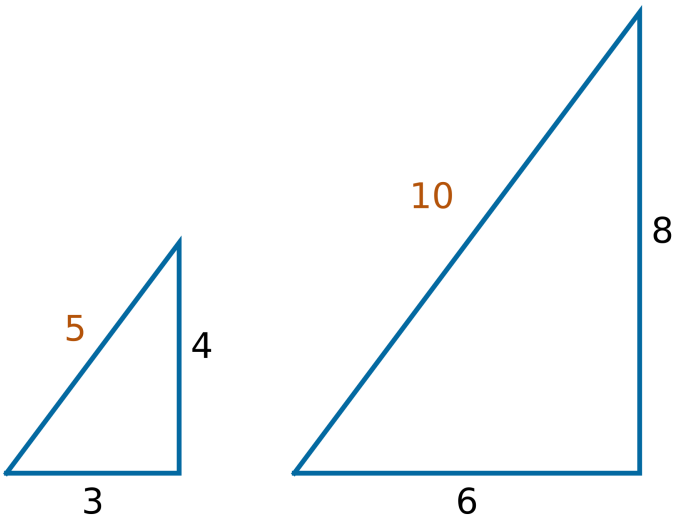
Angle facts

- Angles on a straight line add to 180°; angles around a point add to 360°; vertical angles are equal.
- With parallel lines cut by a transversal, corresponding angles and alternate angles are equal; co-interior angles add to 180°.
- The angles of a triangle add to 180°; an exterior angle equals the sum of the two opposite interior angles.

Similar and congruent triangles

- Triangles are similar if their angles are equal (two pairs are enough). Corresponding sides are in the same ratio k.
- Lengths scale by k, areas by k^2, volumes by k^3.
- Triangles are congruent (same shape and size) by SSS, SAS, ASA, AAS or, for right triangles, hypotenuse–leg. Angles alone fix shape, not size.

Length ×2; area ×4 for similar triangles



Here $k = 2$, so the areas are 6 and 24: a ratio of $2^2 = 4$.

Right triangles

$$a^2 + b^2 = c^2 \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

- 45°–45°–90° sides are in the ratio $1 : 1 : \sqrt{2}$; 30°–60°–90° sides are $1 : \sqrt{3} : 2$.
- $\sin x° = \cos(90 - x)°$, since the two acute angles of a right triangle add to 90°.
- Common whole-number triangles: 3–4–5, 5–12–13 and their multiples.

Area

- Triangle area $= \frac{1}{2} \times$ base $\times$ perpendicular height, where the height is at right angles to the base, not a slanted side.
- Composite shapes: split into rectangles and triangles, or subtract a cut-out.

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Worked example: similar containers

Two similar containers have surface areas 96 cm^2 and 216 cm^2. The smaller holds 160 cm^3.

- Area ratio $= \dfrac{216}{96} = 2.25 = k^2$, so $k = 1.5$.
- Volume ratio $= k^3 = 3.375$.
- Larger volume $= 160 \times 3.375 = 540$ cm^3. Using the area ratio would wrongly give 360 cm^3.

Worked example: a right triangle

A right triangle has hypotenuse 13 cm and one leg 5 cm.

- Other leg: $\sqrt{13^2 - 5^2} = \sqrt{144} = 12$ cm.
- Area $= \frac{1}{2} \times 5 \times 12 = 30$ cm^2.
- For the angle θ opposite the 5 cm leg, $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$.

Common errors

- Using the length ratio for areas or volumes.
- Matching sides of similar triangles in the wrong order.
- Using a slanted side as the height.
- Assuming lines are parallel because they look parallel.
- Assuming equal angles make triangles congruent.

Key terms

similarity: Having the same shape, with all lengths in the same ratio.

scale factor: The number every length is multiplied by.

congruence: Having the same shape and size.

perpendicular height: The height measured at right angles to the base.

Check yourself

- Two angles of a triangle are 50° and 60°. Find the third, and the exterior angle next to it.
- Lengths are multiplied by 3. What happens to area?
- $\sin 30° = \cos$?

Answers: 70°, and $180° - 70° = 110°$. It is multiplied by 9. $\cos 60°$.

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Circle equations, arcs and unit-circle trigonometry

SAT

In 1637 René Descartes published *La Géométrie* as an appendix to his *Discourse on Method*. It showed how to describe a curve by an equation in two coordinates, so that geometry problems could be solved by algebra. A circle becomes the set of points at a fixed distance from a centre, and Pythagoras's theorem turns that into an equation. SAT circle questions use both views: the picture and the equation.

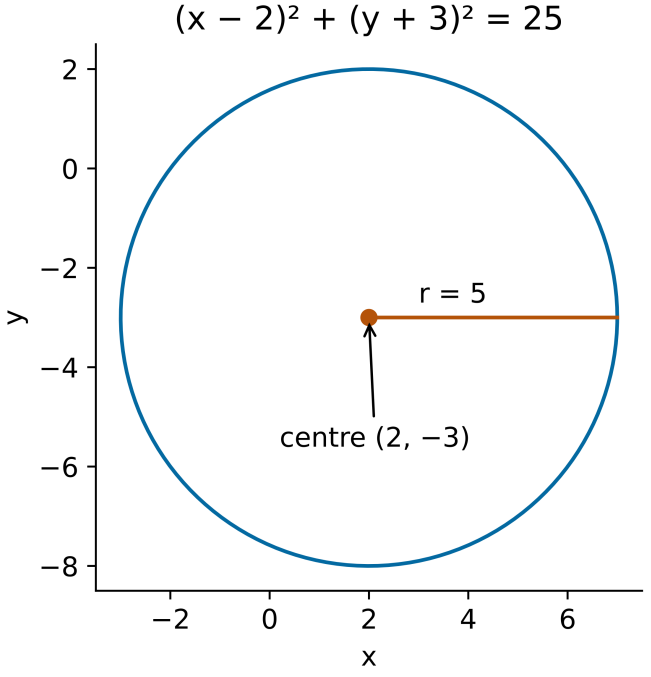
These questions ask you to:

- Read and write a **circle equation** 圆方程, completing the square when needed.
- Convert between degrees and the **radian** 弧度.
- Find **arc length** 弧长 and the area of a **sector** 扇形.
- Use the **tangent** 切线 and inscribed-angle facts, and the unit circle.

The equation of a circle

$$(x-h)^2+(y-k)^2=r^2$$

- The centre is (h,k) and the radius r. Watch the signs: $(y+3)^2$ means $k=-3$.
- From $x^2+y^2+Dx+Ey+F=0$, complete the square in x and in y.
- A point is on the circle if it satisfies the equation, inside if the left side is less than r^2.



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Radians, arcs and sectors

- $180° = \pi$ radians, so multiply degrees by $\dfrac{\pi}{180}$.
- With θ in radians: arc length $s = r\theta$, sector area $A = \frac{1}{2}r^2\theta$.
- In degrees: arc $= \dfrac{\theta}{360} \times 2\pi r$, sector $= \dfrac{\theta}{360} \times \pi r^2$.

Angles and tangents

- An inscribed angle is half the central angle on the same arc. An angle in a semicircle is 90°.
- A tangent touches the circle once and is perpendicular to the radius at that point.
- So the tangent's slope is the negative reciprocal of the radius's slope.

The unit circle

- On a circle of radius 1 centred at the origin, the point at angle θ is $(\cos\theta, \sin\theta)$.
- So $\cos^2\theta + \sin^2\theta = 1$.
- Signs: both positive in quadrant I; cos negative in II and III; sin negative in III and IV.

Worked example: completing the square

Find the centre and radius of $x^2 + y^2 - 8x + 2y + 8 = 0$.

- Group: $(x^2 - 8x) + (y^2 + 2y) = -8$.
- Complete: $(x-4)^2 - 16 + (y+1)^2 - 1 = -8$, so $(x-4)^2 + (y+1)^2 = 9$.
- Centre $(4, -1)$, radius 3. Check $(7, -1)$: $3^2 + 0^2 = 9$, so it is on the circle.

Worked example: a sector

A sector has radius 8 cm and angle 135°.

- Radians: $135 \times \dfrac{\pi}{180} = \dfrac{3\pi}{4}$.
- Arc: $8 \times \dfrac{3\pi}{4} = 6\pi$ cm. Area: $\frac{1}{2} \times 64 \times \dfrac{3\pi}{4} = 24\pi$ cm^2.
- An inscribed angle on the same arc is $135 \div 2 = 67.5°$.

Common errors

- Getting the sign of the centre wrong.
- Giving r^2 as the radius.
- Using degrees in $s = r\theta$.
- Forgetting the 90° between a tangent and a radius.

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- Choosing the wrong sign for cos or sin in a quadrant.

Key terms

circle equation: $(x-h)^2+(y-k)^2=r^2$, for centre (h,k) and radius r.

radian: The angle at the centre made by an arc equal in length to the radius.

arc length: The distance along part of a circle.

sector: The region between two radii and an arc.

tangent: A line that touches a circle at exactly one point.

Check yourself

- Convert 60° to radians.
- Find the arc length for radius 6 and angle $\frac{\pi}{3}$.
- A circle has centre $(0,0)$ and passes through $(3,4)$. What is the slope of the tangent there?

Answers: $\frac{\pi}{3}$. 2π. The radius has slope $\frac{4}{3}$, so the tangent has slope $-\frac{3}{4}$.

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Central ideas, details and the scope of a claim

SAT

In late August 1854 cholera killed more than 500 people in ten days in Soho, London. The doctor John Snow marked each death on a street map and saw them cluster around one water pump on Broad Street. On 7 September he persuaded the local authorities to remove the pump's handle. His central claim was precise: the disease spread through contaminated water. Many officials preferred the idea that it spread through bad air, and Snow's claim was only widely accepted years later. A claim is strongest when it says exactly what the evidence shows: no more, no less.

Central-idea questions ask you to:

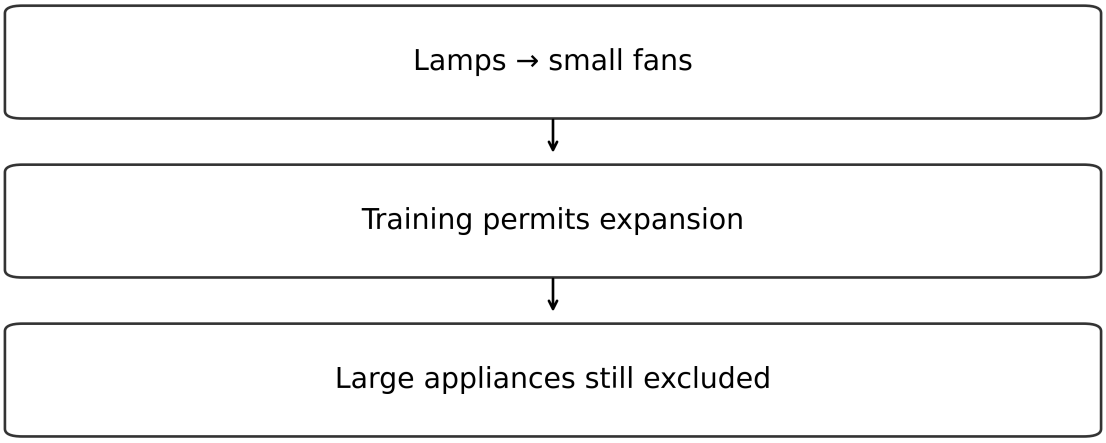
- State the **central idea** 中心思想 of a passage in one sentence.
- Tell a supporting **detail** 细节 apart from the main point.
- Keep the **scope** 范围 of the claim: its group, place, time and strength.
- Reject summaries that the **evidence** 证据 in the passage does not support.

Finding the central idea

- Write a few words for each sentence, then ask what connects them.
- Many SAT passages follow a pattern: a change, a benefit with a limit, a problem with a response, a finding with a caution.
- A good central idea keeps both halves of such a pattern.

Details and their jobs

- A detail gives an example, a reason, a number or a limitation.
- Ask what job a detail does for the central idea: does it explain a benefit, or a limit?
- Two details can do different jobs; do not merge them.



Worked steps: Lamps → small fans → Training permits expansion → Large appliances still excluded

Scope and strength

- Words like “often”, “some”, “may” and “in the trial” set the scope. The right answer keeps them.
- Wrong answers widen the scope (“all”, “every”, “replaced”) or change the strength (“proves”, “never”).
- Wrong answers can also contradict a detail, or add a cause the passage does not give.

Practice passage

Passage. The city archive began a trial in which readers could search handwritten shipping registers using an online index. Volunteers entered names and dates, while staff checked entries that were difficult to read. The index often helped researchers locate a likely page before travelling to the archive. It did not replace the register images: two people with the same name could still be distinguished only by reading nearby entries. Staff therefore described the index as a route into the records rather than a final answer to every question. They also kept a public list of corrected entries so that researchers could see how the index changed.

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Worked example: the central idea

- Sentence summaries: a search trial; volunteers enter, staff check; often helps find a page; does not replace the images; "a route in, not a final answer"; corrections are public.
- Central idea: the index helps researchers find likely records, but the register images are still needed for some questions.
- This keeps both the benefit and the limit, and the word "often".

Worked example: rejecting summaries

- "Volunteers replaced staff." Contradicted: staff checked difficult entries.
- "The index made archive visits unnecessary." Too wide: it "often" helped, and the images are still needed.
- "The archive hid changes to the index." Contradicted: it kept a public list of corrections.
- The detail about two people with the same name explains the limit; the corrections list does a different job, showing how the index changed.

Common errors

- Choosing the most memorable detail as the central idea.
- Keeping the benefit and dropping the limit.
- Missing a scope word such as "often" or "some".
- Accepting a summary that contradicts one sentence.
- Adding a cause or result the passage does not state.

Key terms

central idea: The main point of the whole passage.

detail: A specific fact or example that supports the main point.

scope: How far a claim reaches: which group, place, time, and how strongly.

evidence: Information in the passage that supports a claim.

Check yourself

- A passage says a new bus route "cut most journey times, though not at rush hour." Give its central idea.
- Why is "The route made every journey faster" wrong?
- What job does the phrase "though not at rush hour" do?

Answers: the route shortened most journeys, with a rush-hour limit. "Every" widens the scope beyond "most". It sets a limit on the benefit.

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Choose evidence for a precise textual or numerical claim

SAT

Choosing Evidence for a Textual or Numerical Claim

During the Crimean War, in 1854–1855, the nurse Florence Nightingale kept records at the British army hospital at Scutari. Her figures showed that far more soldiers were dying of preventable diseases such as cholera and typhus than of their wounds. In 1858 she published diagrams, later called "rose diagrams", that showed deaths month by month by cause. She chose death rates, not just counts, so that months with different numbers of soldiers could be compared fairly. SAT evidence questions ask you to make the same choice: which piece of information actually supports this claim?

These questions ask you to:

- Choose the quotation that best supports a claim about a text.
- Choose the data from a table or graph that supports a claim.
- Check the **relevance** 相关性 of each option to the exact claim.
- Use the right **comparison base** 比较基准, counts or percentages, and calculate a change in a **percentage point** 百分点.

Restate the claim first

- Before reading the options, put the claim in your own words, with every part: who, what, and how much.
- The right evidence supports every part. An option that supports half of the claim is wrong.
- A true and interesting quotation can still be irrelevant.

Numbers: counts, rates and changes

- A count says how many; a rate or percentage says how many out of how many. Groups of different sizes need rates.
- A change from 40% to 70% is 30 percentage points, which is a 75% relative increase. Do not mix the two.
- "More successes" and "a larger improvement" are different claims and need different numbers.

Original hypothetical data: choose the claim's comparison

Group	Learners	Before	After	Correct (%)
A	20	12	16	60% → 80%
B	40	24	28	60% → 70%

In the figure, B has more correct answers after (28 against 16), but A improved more: 20 points against 10.

Evidence and cause

- A before-and-after comparison shows a change, not its cause. Practice, a different task or other events may explain it.
- Evidence for a causal claim usually needs a comparison group, ideally with random assignment.

Practice study

Hypothetical study. Two groups used a revised catalogue for a second search session. No random assignment is reported. Group P had 30 users, with 12 successful searches before and 21 after. Group Q had 50 users, with 30 successful searches before and 35 after. Each user completed one search in each session. The researchers claimed that P showed the larger percentage-point improvement, while Q still had more successful searches in the second session. A separate comment said the revision must have caused every improvement.

Worked example: matching numbers to claims

- P: $\frac{12}{30} = 40\%$ before, $\frac{21}{30} = 70\%$ after: up 30 points.
- Q: $\frac{30}{50} = 60\%$ before, $\frac{35}{50} = 70\%$ after: up 10 points.
- "P improved more" is supported by 30 points against 10. "Q had more successes" is supported by 35 against 21.
- Swapping them, for example using 35 against 21 to support "Q improved more", would confuse a final count with a change.

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Worked example: a claim that goes too far

- The comment says the revision "must have caused every improvement".
- No random assignment or comparison group is reported, and users were simply searching for a second time, so practice could explain part of the change.
- Evidence that would help: a matched group using the old catalogue in both sessions. It would help test the cause, not prove it alone.

Common errors

- Choosing evidence that is accurate but supports a different claim.
- Comparing counts for groups of different sizes.
- Confusing percentage points with per cent change.
- Using a before-and-after change as proof of a cause.
- Choosing evidence for only half of the claim.

Key terms

relevance: How directly a piece of information bears on the claim.

comparison base: What a number is compared with or out of, such as a group total.

percentage point: The unit for the difference between two percentages.

Check yourself

- A rate rises from 20% to 25%. What is the change in percentage points, and the relative change?
- A claim says "Town X has a higher recycling rate than Town Y." Is "X recycled 500 tonnes and Y 300" good evidence?
- A quotation is true but describes a different group from the claim. Can it support the claim?

Answers: 5 percentage points; 25% relative. No: the towns may have different populations, so rates are needed. No: it is not relevant.

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Complete a passage with a warranted inference

SAT

Completing a Passage with a Warranted Inference

In 1747 James Lind, a surgeon on HMS *Salisbury*, took twelve sailors with scurvy and divided them into six pairs. Each pair received a different treatment: cider, vinegar, sea water, a spicy paste, dilute sulphuric acid, or two oranges and a lemon a day. The pair given citrus fruit recovered fastest, and one was fit for duty within six days. Lind published his results in 1753. With only two sailors per treatment, he could reasonably say citrus fruit helped; he could not say why. SAT inference questions reward the same caution.

Inference questions ask you to:

- Complete a passage with the **inference** 推断 that follows most logically.
- Choose a **warranted inference** 有依据的推断, one the **evidence** 证据 in the passage supports.
- Keep any **qualification** 限定 the passage includes, such as a small or unequal sample.

What makes an inference warranted

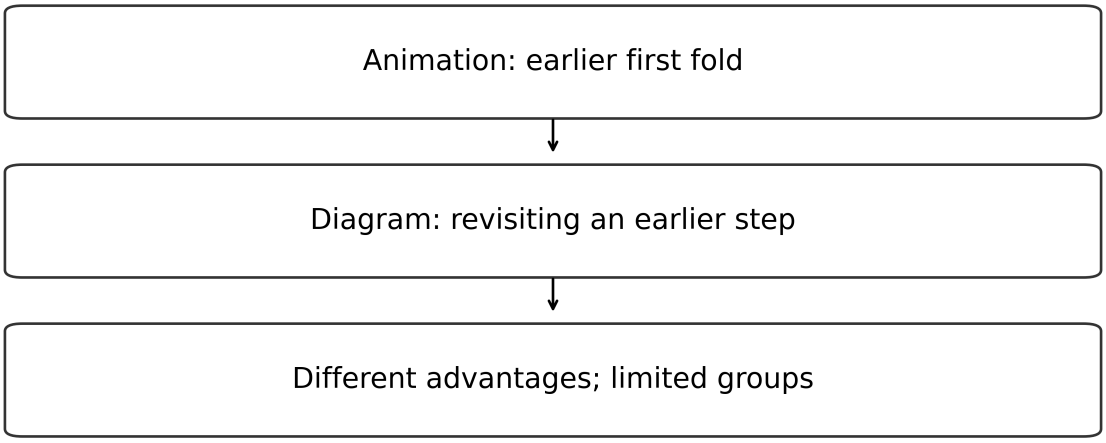
- It joins facts stated in the passage. It does not need outside knowledge.
- It keeps the passage's scope: the same groups, the same task, the same strength.
- It often sounds modest: "may", "in this case", "for some".

Testing the options

- For each option, ask: which sentences support it? If none, reject it.
- Reject options that turn a relative finding ("more often", "sooner") into an absolute one ("always", "never", "useless").
- Reject options that add a new cause, group or time.
- Read the sentence with the blank: the conclusion must fit its first words, such as "therefore" or "this suggests".

How limits change an inference

- Small groups, groups that may differ, or no random assignment weaken a general conclusion.
- A later study with different results weakens a general claim; it does not show the first study was false.



Worked steps: Animation: earlier first fold → Diagram: revisiting an earlier step → Different advantages; limited groups

Practice passage

Passage. A workshop offered a printed diagram and an animated demonstration of a folding process. Participants using the animation began the first fold sooner on average. However, when asked to repair a mistake later, participants using the diagram more often returned to the correct earlier step without help. The groups were small, and the organisers did not report whether participants had equal experience. They concluded that choosing a teaching aid should take account of the stage of the task.

Worked example: completing the conclusion

- Facts: animation users started sooner; diagram users more often fixed mistakes alone; groups were small and may have differed.
- Warranted inference: the animation may help people begin, while the diagram may help them go back to an earlier step, though the small groups limit this.
- This matches the organisers' own conclusion about the "stage of the task".

Worked example: rejecting overstatements

- "Animations prevent mistakes." Unsupported: the study measured fixing mistakes, not avoiding them.
- "Diagrams are useless for starting." Unsupported: diagram users started later on average, not never.
- If a new study with equal experience found equal correction rates, it would weaken a general claim that diagrams are better, and suggest experience may explain the first result.

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Common errors

- Choosing an option that is true in real life but not supported by the passage.
- Turning "sooner" or "more often" into "always" or "never".
- Ignoring the limit stated in the passage.
- Adding a reason the passage never gives.
- Choosing a conclusion that does not fit the words before the blank.

Key terms

inference: A conclusion drawn from information that is stated.

warranted inference: An inference that the evidence actually supports.

evidence: The facts in the passage that support a conclusion.

qualification: A limit added to a claim to keep it accurate.

Check yourself

- A passage says students who slept more scored higher, in one class of 25. Is "Sleep raises scores for all students" warranted?
- Which word in an option should make you check the scope: "may" or "always"?
- A passage gives a finding and then "however, the sample was small." What should the right conclusion include?

Answers: no: it widens the scope and claims a cause. "Always". A qualification, such as "may" or "in this group".

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Words in context: meaning, polarity and collocation

SAT

Words in Context: Meaning, Tone and Collocation

In April 1755 Samuel Johnson published his *Dictionary of the English Language*, the result of about nine years' work with a small team of assistants. It defined more than 40,000 words, and Johnson illustrated their meanings with over 100,000 quotations from earlier writers. He knew a definition alone was not enough: a word means what it means in a sentence. SAT vocabulary questions test exactly that. You are not asked for a word's dictionary meaning but for its meaning in this passage.

Words-in-context questions ask you to:

- Find the **word in context** 语境中的词义: the meaning a word has in this sentence.
- Use clues to decide the **contextual meaning** 语境义, including positive or negative tone.
- Recognise **collocation** 词语搭配, the words that naturally go together.
- Notice **connotation** 内涵意义, the feeling a word carries.

Predict before you look

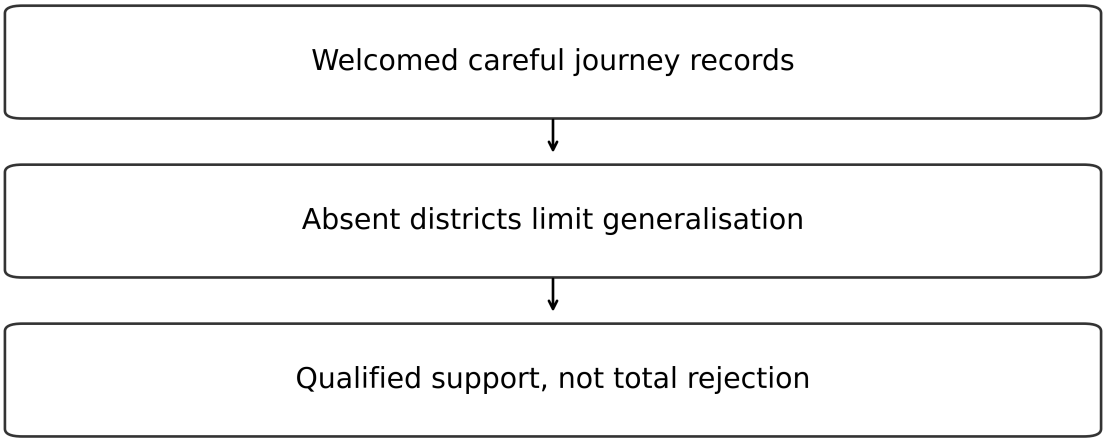
- Cover the word and say what the sentence needs, in simple words.
- Use clues nearby: an example, a contrast ("but", "although"), a cause ("because"), or a restatement after a colon.
- Then choose the option closest to your prediction.

Tone and connotation

- Decide whether the context is positive, negative or neutral. "Welcomed" is positive; "cautioned" signals concern.
- Two words can share a meaning but differ in feeling: "thrifty" is positive; "stingy" is negative.
- An option with the right meaning but the wrong tone is wrong.

Common words with uncommon meanings

- The SAT often uses everyday words in a less common sense: "qualified" can mean limited; "reservation" can mean a doubt; "check" can mean stop.
- Test the familiar meaning in the sentence; if it does not fit, look for another.
- Collocation helps: we give "qualified support" and "express a reservation".



Worked steps: Welcomed careful journey records → Absent districts limit generalisation → Qualified support, not total rejection

Practice passage

Passage. The reviewers welcomed the survey's careful record of individual journeys, but they gave its city-wide conclusions only qualified support. The routes had been chosen for convenience, and several districts were absent. One reviewer called the record illuminating: it made previously overlooked travel difficulties easier to understand. Another cautioned that extending the conclusions would require a broader sample. The authors accepted this reservation and proposed a second survey.

Worked example: three words

- "Qualified": limited, conditional. Clue: the routes were convenient and districts were missing.
- "Illuminating": making something clearer. Clue: the colon, then "easier to understand".
- "Reservation": a doubt or concern. Clue: it refers back to the caution about extending conclusions; it is not a booked seat.

Worked example: testing replacements

Replace "illuminating" with *revealing*, *dazzling* or *visible*.

- *Revealing* keeps the meaning: the record makes difficulties understood.
- *Dazzling* adds admiration and brilliance that the passage does not express.
- *Visible* means "able to be seen", not "explaining". So *revealing* is best here. The others are not always wrong, only wrong in this sentence.

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Common errors

- Choosing the most familiar meaning of the word.
- Ignoring a contrast word such as “but”.
- Picking a word with the right meaning but the wrong tone.
- Choosing a word that does not collocate with the words around it.
- Deciding before reading the whole sentence.

Key terms

word in context: The meaning a word has in a particular sentence.

contextual meaning: The sense a word takes from the words around it.

collocation: A pair or group of words that are often used together.

connotation: The feeling or attitude a word suggests beyond its basic meaning.

Check yourself

- “The new rule will check the spread of the weed.” What does “check” mean?
- Which is more positive about a careful spender: “frugal” or “miserly”?
- “Her account of the trip was spare: only dates and places.” What does “spare” mean here?

Answers: limit or stop. “Frugal”. Brief, with little detail.

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Text structure, sentence function and rhetorical purpose

SAT

In 1931 Harry Beck, an engineering draughtsman who had been laid off by the London Underground, drew a new map of the network in his spare time. The old maps followed real geography, so stations in central London were crowded together and outer lines ran off the edge. Beck used only straight lines, at 45° and 90°, and spaced the stations evenly, like an electrical circuit. The company was doubtful, but printed a trial run in 1933; passengers liked it, and Beck kept improving it for years. His story has a clear structure: a problem, a proposed solution, a trial, a limit and a refinement. SAT structure questions ask you to see that shape in a passage.

These questions ask you to:

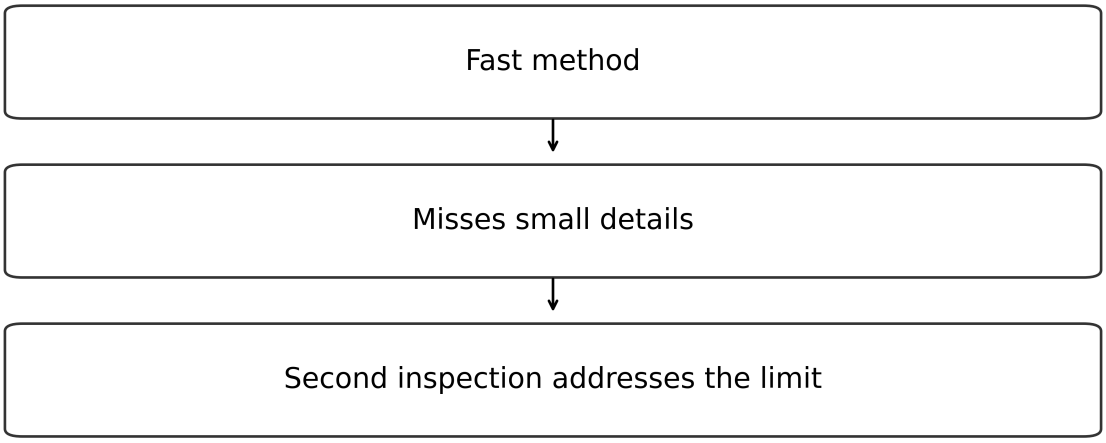
- Describe the overall structure of a text as a sequence of actions.
- Identify the **sentence function** 句子作用 of one sentence or phrase.
- State the text's **rhetorical purpose** 修辞目的.
- Tell a reported **proposal** 提案 from the writer's own **argument** 论证, and notice a **qualification** 限定.

Describing structure with verbs

- Describe what each part does, not what it is about: "introduces a problem", "reports a solution", "qualifies a result", "describes a refinement".
- Common shapes: problem → solution; claim → evidence → limit; old view → new finding; general → example.
- The right answer follows the order of the passage.

The function of a sentence

- Ask what would be lost if the sentence were removed: an example, a reason, a contrast, a limit?
- Words like "however", "although" and "instead" often mark the function: a contrast or a qualification.
- A qualifying phrase limits a claim; it does not reverse it.



Worked steps: Fast method → Misses small details → Second inspection addresses the limit

Purpose and point of view

- The purpose is usually a verb phrase: “to describe how a design was improved”, “to challenge an assumption”.
- Separate what others proposed or believed from what the writer states.
- A passage that reports a proposal does not necessarily support it.

Practice passage

Passage. Early walking maps marked every route with the same solid line. This kept the map simple, but walkers could not tell whether a route included steps. Some designers proposed adding a written note beside every staircase. A trial instead used a distinct line pattern for routes without steps and explained it in a clear legend. Walkers then identified these routes more accurately in the trial, although some readers needed help noticing the legend. The designers retained the pattern and made the legend more prominent.

Worked example: the overall structure

- Sentence 1 describes an earlier design; sentence 2 names its problem.
- Sentence 3 reports one proposal; sentence 4 a different solution that was tested.
- Sentence 5 gives the result, with a qualification; sentence 6 describes a refinement.
- Purpose: to describe how a map design was improved through a trial.

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Worked example: function and point of view

- "Although some readers needed help noticing the legend" qualifies the improved accuracy by naming a remaining problem. It does not show the trial failed: accuracy still improved.
- Does the writer recommend written notes beside every staircase? No. That is a reported proposal; the design tested and retained was the line pattern.
- An answer that says "the writer argues for written notes" mixes up two stages of the story.

Common errors

- Describing the topic when the question asks for the function.
- Treating a qualification as a reversal.
- Giving a reported proposal to the writer.
- Choosing a structure whose order does not match the passage.
- Naming a purpose that fits only one sentence.

Key terms

sentence function: The job a sentence does in a text, such as giving an example or a limit.

rhetorical purpose: What the writer is trying to do with the whole text.

proposal: A suggested plan or solution.

argument: A claim supported by reasons.

qualification: A phrase that limits a claim.

Check yourself

- A passage gives an old belief, then a study that contradicts it. What is the function of the study?
- "The plan cut costs, although it took longer." What does the "although" clause do?
- A passage says "Some critics suggested closing the road." Does the writer support closing it?

Answers: it challenges the old belief. It qualifies the benefit. Not shown: the writer only reports the suggestion.

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Compare two texts without merging their claims

SAT

Comparing Two Texts Without Merging Their Claims

In the early 1860s the French scientists Louis Pasteur and Félix Pouchet argued about where the microbes in spoiled broth came from. Both men saw the same thing: boiled broth, left standing, soon filled with tiny living things. Pouchet argued that they arose from the broth itself; Pasteur argued that they came from dust in the air. Pasteur boiled broth in flasks with long, curved "swan necks" that let air in but trapped dust, and no microbes grew. The two agreed on the observation and disagreed about its cause. Many SAT paired-text questions have exactly this shape.

These questions ask you to:

- Make a **cross-text comparison** 跨文本比较 of two short texts.
- Find where they agree, and the precise point where they disagree.
- Predict how one author would respond to the other's **interpretation** 解释 or **evidence** 证据.
- Recognise when an author offers an **alternative explanation** 其他解释.

Summarise each text separately

- Write one sentence for Text 1 and one for Text 2 before comparing them.
- Note each author's claim, evidence and attitude: certain, cautious, critical.
- Keep the authors apart: do not give one author's reason to the other.

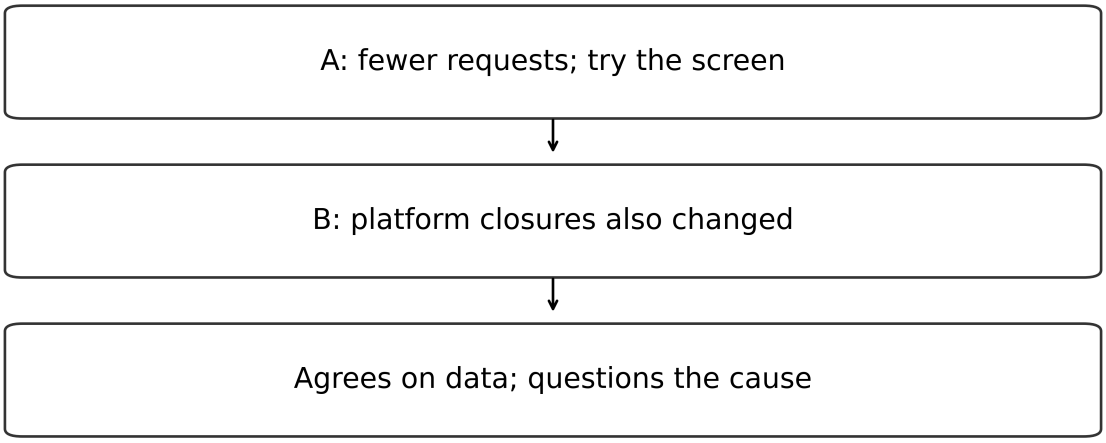
Agreement and disagreement

- Two texts often agree on the facts and disagree on what they mean.
- Disagreement can be partial: "yes, but only under these conditions" or "possibly, but there is another cause".
- The right answer names the exact point of difference, not a general "they disagree".

Predicting a response

- Ask: what in Text 2 bears on Text 1's claim? A new fact, a limit, a different cause?
- Check every part of the claim you are testing: the size, the cause and the scope.
- A cautious author rarely "rejects" anything outright; choose an answer with matching strength.

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Worked steps: A: fewer requests; try the screen → B: platform closures also changed → Agrees on data; questions the cause

Practice texts

Text A. During a two-week trial, a station displayed route information on a large screen. Requests for platform directions fell from 60 in the preceding fortnight to 36. The screen seems a promising way to help travellers find their platforms, and other stations should consider a trial.

Text B. Fewer requests were recorded during the screen trial, but several platforms were closed for repairs at the same time. The remaining journeys may have been easier to navigate. Before treating the screen as the explanation, compare similar periods with the same platform arrangement. A further trial is reasonable if it separates these changes.

Worked example: agreement and disagreement

- Text A: requests fell, so the screen looks promising and should be tried elsewhere.
- Text B: requests fell, but platform closures happened at the same time, so the cause is unclear.
- Agreement: requests fell, and a further trial is reasonable.
- Disagreement: whether the screen explains the fall. Text B offers an alternative explanation; it does not say the screen fails.

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Worked example: testing a strong claim

How would the author of Text B respond to "The screen caused a 40% reduction at every station"?

- The size is right for one station: $\dfrac{60 - 36}{60} = 40\%$.
- "Caused": B questions this, because of the closures.
- "Every station": there are no data from other stations. So B would reject the claim's certainty and scope, not its arithmetic.
- Useful further evidence: similar periods with the same platforms open, counting travellers as well as requests.

Common errors

- Merging the two texts into one view.
- Saying the authors "completely disagree" when they share the facts.
- Choosing a response that is stronger than the author's tone.
- Testing only one part of a claim.
- Giving Text 2's evidence to Text 1's author.

Key terms

cross-text comparison: Comparing the claims, evidence and attitudes of two texts.

interpretation: An explanation of what facts mean.

evidence: Facts or data offered to support a claim.

alternative explanation: A different possible cause for the same observation.

Check yourself

- Text 1 says a diet improved health. Text 2 says participants also exercised more. What does Text 2 offer?
- Is "Text 2 rejects Text 1's data" a good description in that case?
- What should you write before comparing two texts?

Answers: an alternative explanation. No: it accepts the data and questions the cause. A one-sentence summary of each.

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Use research notes for the requested rhetorical goal

SAT

Using Research Notes for the Requested Rhetorical Goal

In 1962 the biologist Rachel Carson published *Silent Spring*, about the effects of pesticides such as DDT on birds, fish and people. Carson had spent years collecting scientific reports, many of them technical. Her skill was choosing which findings to use, and how to present them, for readers who were not scientists. The book helped lead to a ban on most uses of DDT in the United States in 1972. SAT synthesis questions give you research notes and a goal; your job, like Carson's, is to choose and combine what serves that goal.

These questions ask you to:

- Use **rhetorical synthesis** 修辞综合: combine notes into one sentence.
- Meet the stated **rhetorical goal** 修辞目标 exactly.
- Emphasise a **similarity** 相似 or a **contrast** 对比 when the goal asks.
- Leave out any **detail** 细节 the goal does not need.

Read the goal first

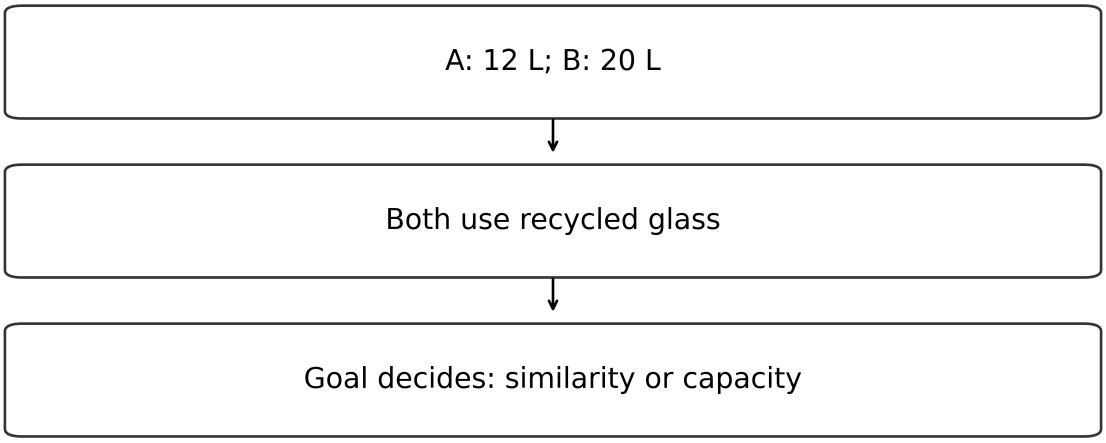
- The question ends with a goal, such as "emphasise a similarity", "present the study to an audience unfamiliar with it" or "explain the advantage of a method".
- Underline its key words. The right option does exactly that.
- Read the notes after the goal, looking for what it needs.

Use only the notes

- **Synthesis** 综合 combines notes; it does not add facts, causes or opinions.
- Watch for options that join two true notes with an unsupported "because" or "so".
- If the notes give totals, do not turn them into counts of separate items.

Matching options to goals

- Similarity: look for "both", "also", "like".
- Contrast: look for "whereas", "while", "unlike", "but".
- Introducing to a new audience: the option must explain what the thing is.
- An accurate, well-written option that meets a different goal is wrong.



Worked steps: A: 12 L; B: 20 L → Both use recycled glass → Goal decides: similarity or capacity

Practice notes

Original research notes:

- The Bay project restored 3 km of walking paths in 2024.
- The Hill project restored 5 km of walking paths in 2024.
- Both projects used volunteer teams.
- Bay used local stone for boundary markers.
- Hill used recycled timber for boundary markers.
- The notes give no costs, visitor counts or durability measurements.

Worked example: two goals, two sentences

- Goal: emphasise a similarity in staffing. "Both Bay and Hill used volunteer teams."
- Goal: emphasise the difference in distance. "Hill restored 5 km of walking paths, 2 km more than Bay."
- Goal: contrast the marker materials. "Bay used local stone for its boundary markers, whereas Hill used recycled timber." The shared year 2024 is left out, since it does not serve this goal.

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Worked example: an unsupported link

A student writes: "Hill restored more paths because recycled timber lasts longer."

- "Because": the notes give no reason for the difference in distance.
- "Lasts longer": the notes give no durability measurements.
- "More paths": the notes give total kilometres, not a number of paths.
- A supported version: "Hill restored a longer total distance and used recycled timber for its markers."

Common errors

- Choosing an accurate option that meets a different goal.
- Adding a cause the notes do not give.
- Including details the goal does not need.
- Forgetting to explain a thing when the audience is unfamiliar with it.
- Changing the numbers' meaning, such as kilometres into "paths".

Key terms

rhetorical synthesis: Choosing and combining information from notes to achieve a purpose.

rhetorical goal: The purpose a sentence must achieve, as stated in the question.

similarity: A feature that two things share.

contrast: A difference between two things.

detail: A specific piece of information.

synthesis: Combining information from several sources.

Check yourself

- The goal is "emphasise a difference". Which word suits it: "both" or "whereas"?
- Notes say a museum opened in 1990 and has 2 million visitors a year. Is "The museum is popular because it is old" supported?
- The goal is to introduce a study to readers who do not know it. What must the sentence include?

Answers: "whereas". No: the notes give no cause. What the study is: who did it and what it examined.

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Transitions: choose the logical relationship first

SAT

On 19 November 1863 Abraham Lincoln spoke for about two minutes at the dedication of a soldiers' cemetery at Gettysburg, Pennsylvania. His speech had about 270 words. Halfway through, after saying that it was right to dedicate the field, he turned with one word: "But, in a larger sense, we can not dedicate, we can not consecrate, we can not hallow this ground." That "But" changed the speech from a ceremony into a call to the living. A **transition** 衔接词 tells the reader how one idea relates to the next, and on the SAT you must choose it by that relationship, not by its sound.

Transition questions ask you to:

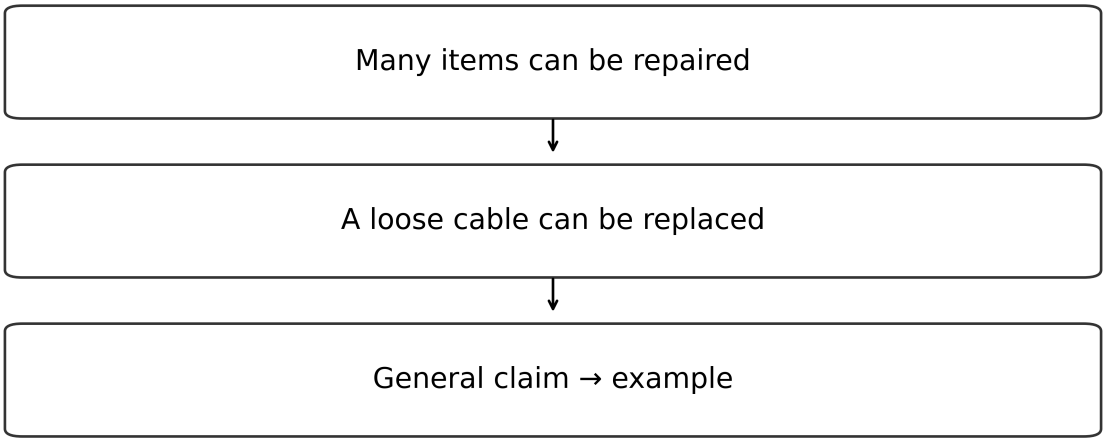
- Decide the relationship between two sentences before reading the options.
- Choose a transition for addition, **contrast** 对比, **consequence** 结果, example or sequence.
- Recognise a **concession** 让步: admitting a point before making another.
- Use "however" with correct sentence boundaries.

Name the relationship first

- Read the sentence before the blank and the sentence after it. Cover the options.
- Ask: is the second idea more of the same, a difference, a result, an example, a later step or a concession?
- Only then look for a transition that expresses that relationship.

Families of transitions

- Addition: moreover, furthermore, in addition, also.
- Contrast: however, by contrast, nevertheless, on the other hand.
- Consequence: therefore, as a result, consequently, thus.
- Example: for example, for instance. Sequence: then, next, finally, subsequently.
- Concession: admittedly, granted, of course (often followed later by "but" or "still").
- Similarity: similarly, likewise.



Worked steps: Many items can be repaired → A loose cable can be replaced → General claim → example

Transitions and punctuation

- Transitions such as “however” and “therefore” are not conjunctions: they cannot join two sentences with a comma alone.
- Correct: “It is free; however, it is only open on weekdays.” Or: “It is free. However, it is only open on weekdays.”
- Wrong: “It is free, however it is only open on weekdays.”

Practice passage

Passage. The new catalogue can be searched from home. [1] Readers no longer need to travel merely to find a likely shelf number. It still omits several collections. [2] A reader looking for an unlisted item must contact staff. Some entries also use abbreviations. [3] “MS” identifies a manuscript rather than a printed book.

Worked example: three blanks

1 Searching from home leads to not needing to travel: consequence. “Therefore,” fits.

2 Missing collections lead to contacting staff: consequence again. “Consequently,” fits.

3 “MS” is one case of an abbreviation: example. “For example,” fits; “similarly” would wrongly suggest a second, parallel point.

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Worked example: contrast and concession

- To contrast home access with incomplete coverage: "The catalogue can be searched from home; however, it still omits several collections."
- "The guide is free. _____, it is only available on weekdays." The second idea limits an advantage: contrast ("However"), not consequence. Being free does not cause the weekday timetable.
- A concession version: "Admittedly, the guide is only available on weekdays. Still, it is free."

Common errors

- Choosing a transition by its sound before naming the relationship.
- Using "therefore" when the second idea is not a result.
- Using "similarly" for an example.
- Joining two independent clauses with "however" and a comma alone.
- Reading only the sentence before the blank.

Key terms

transition: A word or phrase that shows how two ideas are related.

contrast: A difference between two ideas.

consequence: A result that follows from something else.

concession: Admitting that an opposing point is true before continuing.

Check yourself

- "The road was icy. _____, the school opened late." Which relationship and which transition?
- "Many birds migrate. _____, the Arctic tern flies from pole to pole." Which transition?
- Correct the punctuation: "It rained, however we played."

Answers: consequence: "As a result". "For example". "It rained; however, we played."

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Sentence boundaries, colons and nonessential information

SAT

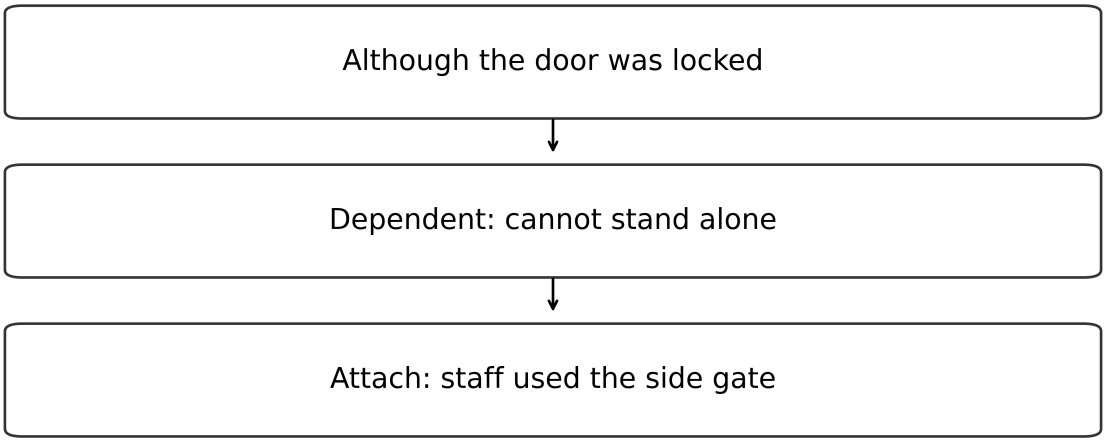
In 2017 a US federal appeals court in Boston decided a case about a missing comma. A Maine law listed tasks for which workers did not get overtime pay: “the canning, processing, preserving, freezing, drying, marketing, storing, packing for shipment or distribution of” foods. Delivery drivers for Oakhurst Dairy argued that, with no comma before “or distribution”, the law excluded “packing”, not “distribution”, so they were owed overtime. The court agreed, and in 2018 the dairy settled for 5 million dollars. Punctuation decides meaning, and the SAT tests the rules that make it clear.

These questions ask you to:

- Mark a **sentence boundary** 句子边界 between two **independent clauses** 独立分句.
- Avoid a **comma splice** 逗号拼接错误 and a run-on sentence.
- Use a **colon** 冒号 only after a complete clause.
- Set off a **nonessential element** 非限定成分 with a matching pair of marks.

The clause test

- An independent clause has a subject and a verb and can stand alone: “Its clock still worked.”
- A dependent clause starts with a word like “although”, “because”, “when” or “which” and cannot stand alone.
- Test each side of the punctuation mark: can it stand alone as a sentence?



Worked steps: Although the door was locked → Dependent: cannot stand alone → Attach: staff used the side gate

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Joining or separating clauses

- Two independent clauses: full stop; semicolon; or comma plus and, but, or, so, yet, for, nor.
- An introductory dependent clause is followed by a comma: “Although the station was closed, its clock still worked.”
- “However” and “therefore” need a semicolon or full stop before them when they start a new clause.

Colons and dashes

- A colon must follow a complete independent clause. After it comes a list, an explanation or an example.
- Wrong: “One part needed: a replacement.” The verb “needed” has not got its object yet.
- A single dash can do the same job as a colon; a pair of dashes marks an interruption.

Essential and nonessential information

- Nonessential information adds detail but does not identify which one: “The clock, which stood above the entrance, had become a meeting point.”
- Mark it with two commas, two dashes or two brackets: never one of each.
- Essential information identifies the noun and takes no commas: “The clock that stood above the entrance was restored; the other was not.”

Practice passage

Editing passage. Although the old station was closed [1] its clock still worked. The mechanism was simple [2] however, repairing the hands required a specialist. One part needed replacement [3] the narrow axle behind the dial. The clock [4] which stood above the entrance [5] had become a meeting point. For this exercise there is one clock, and the which-clause adds nonessential information.

Worked example: five marks

1 Comma: “Although the old station was closed” is an introductory dependent clause.

2 Semicolon (or a full stop and “However”): both sides are independent clauses.

3 Colon: “One part needed replacement” is complete, and the axle explains it.

4 and [5] Commas (or two dashes): the which-clause is nonessential, since there is only one clock.

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Worked example: agreement inside punctuation

Repair "The clock, above the entrance, and the bell was restored."

- The subject is "the clock … and the bell": two things, so the verb is "were".
- "The clock above the entrance and the bell were restored."
- Commas around "above the entrance" are possible only if it is extra information; they do not change the agreement.

Common errors

- Joining two independent clauses with only a comma.
- Putting a colon after a verb or a preposition.
- Opening nonessential information with a comma and closing it with a dash.
- Putting commas round essential information.
- Missing the comma after an introductory dependent clause.

Key terms

sentence boundary: The point where one complete sentence ends and the next begins.

independent clauses: Groups of words with a subject and a verb that can stand alone as sentences.

comma splice: Two independent clauses joined by a comma alone.

colon: A mark (:) that follows a complete clause and introduces an explanation or list.

nonessential element: Information that adds detail but is not needed to identify the noun.

Check yourself

- Correct: "The bus was late, we walked."
- Is this correct: "She bought: apples, pears and plums."?
- Punctuate: "My brother who lives in Shanghai is a nurse." (I have only one brother.)

Answers: "late; we walked", "late. We walked" or "late, so we walked". No: remove the colon, or write "She bought three fruits: apples, pears and plums." "My brother, who lives in Shanghai, is a nurse."

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Agreement, verb forms, modifiers and logical parallelism

SAT

Agreement, Verb Forms, Modifiers and Parallelism

In 47 BC Julius Caesar defeated King Pharnaces II of Pontus at the Battle of Zela, in what is now Turkey, after a campaign of only a few days. According to the historian Plutarch, he reported the victory to Rome in three words: *Veni, vidi, vici*, "I came, I saw, I conquered." The phrase is remembered partly because of its form: three verbs, the same tense, the same person, the same length. That is parallelism, and with agreement and clear modifiers it makes up the second half of SAT Standard English Conventions.

These questions ask you to:

- Make a **finite verb** 限定动词 agree with its subject, and choose the right tense.
- Make a pronoun agree with its **antecedent** 先行词.
- Place a **modifier** 修饰语 next to the word it describes, and avoid a **dangling modifier** 垂悬修饰语.
- Keep lists and comparisons in **parallelism** 平行结构.

Subject–verb agreement

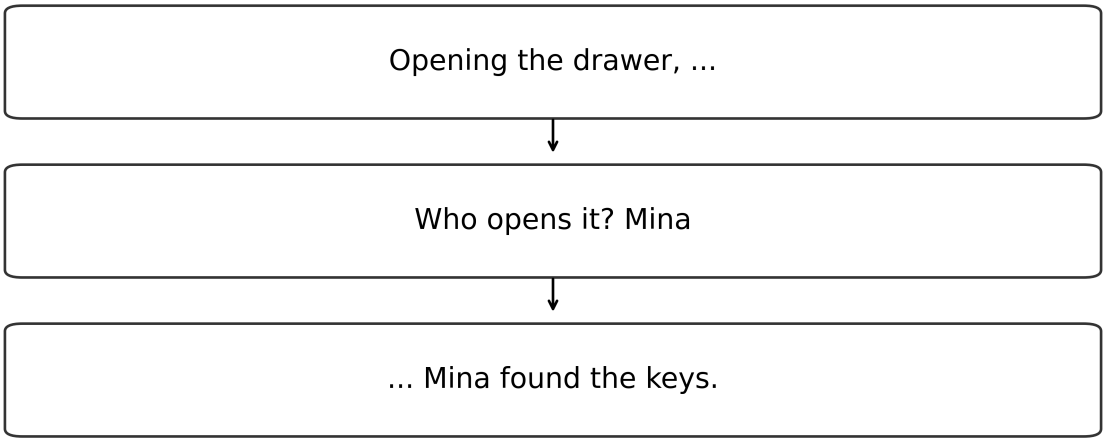
- Find the true subject; ignore phrases in between: "The collection of donated tools includes three drills."
- "Each", "every", "either", "neither" and collective nouns such as "team" usually take a singular verb.
- Two subjects joined by "and" take a plural verb.

Tense and verb form

- Keep tenses consistent unless the time changes. Time words help: "yesterday" needs the past.
- The past perfect ("had checked") shows that one past event happened before another; use it when the order needs to be clear.
- After "to" or a modal ("can", "must"), use the base form.

Pronouns and modifiers

- A pronoun must have one clear antecedent and agree with it: “The team presented its plan.”
- If “she” could mean two people, use a name.
- A modifier at the start of a sentence describes the subject that comes right after the comma.



Worked steps: Opening the drawer, … → Who opens it? Mina → … Mina found the keys.

Parallelism and logical comparison

- Items in a list take the same form: “to clean, label and display”, not “to clean, labelling and display”.
- Compare like with like: “The museum’s hours are longer than the archive’s hours” (or “the archive’s”), not “than the archive”.

Practice passage

Editing passage. The collection of donated tools [1: include/includes] three old drills. Examining the smallest drill, [2: a crack surprised Jo/Jo noticed a crack]. Yesterday Jo [3: repairs/repaired] the casing. The volunteer team plans to clean, [4: label/labelling], and display the tools. The museum’s opening hours are longer than [5: the archive/the archive’s opening hours].

Worked example: five choices

1 "Includes": the subject is "collection", singular.
2 "Jo noticed a crack": Jo is the one examining, so Jo must follow the comma.
3 "Repaired": "yesterday" needs the past.
4 "Label": parallel with "clean" and "display" after "to".
5 "The archive's opening hours": hours are compared with hours, not with a building.

Worked example: unclear reference and sequence

- "When Jo met Mina, she showed her the repaired drill." "She" could be either person. Clear: "When they met, Jo showed Mina the repaired drill."
- "Before the shop opened last Monday, Jo had checked the drills." The past perfect shows the checking came first. Here "before" already shows the order, so the simple past would also be acceptable; judge each sentence.
- A parallel list: "checking cables, replacing screws and cleaning handles", all -ing forms.

Common errors

- Matching the verb to the nearest noun instead of the subject.
- Using "their" for a singular collective noun such as "the team".
- Leaving an opening modifier with no subject it can describe.
- Mixing forms in a list.
- Comparing a thing with a person or place.

Key terms

finite verb: A verb with tense that can be the main verb of a clause.

antecedent: The noun that a pronoun refers back to.

modifier: A word or phrase that describes another part of the sentence.

dangling modifier: A modifier with no clear word in the sentence to describe.

parallelism: Using the same grammatical form for items that are listed or compared.

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Check yourself

- Choose: “Neither of the answers _____ correct.” (is / are)
- Correct: “Walking to school, the rain started.”
- Make parallel: “She likes swimming, to run and cycling.”

Answers: “is”. “Walking to school, I felt the rain start” (or another subject who walks). “She likes swimming, running and cycling.”

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