

Exercise walk-through

16

PSDA-conditional · Probability from a table and a restricted group

SAT · SAT · SAT

16 Probability from a table and a restricted group

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS conditional probability • independent events

Objective

- Calculate event and complement probabilities from counts
- Use the conditioning group as the denominator
- Distinguish independence from disjointness

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

A probability based on equally represented observations is favourable count divided by the relevant total. A two-way table has joint counts in cells, group totals on the margins and a grand total. Read whether the question asks about a joint event, either event or

one event given another before choosing the denominator.

For $P(A \text{ given } B)$, restrict attention to the B group and divide the A-and-B count by the B total. It is not generally equal to $P(B \text{ given } A)$, because the conditioning groups differ. A fraction with the correct numerator but the wrong population still answers the wrong question.

The complement probability is $1-P(A)$. For either A or B, adding separate probabilities double-counts overlap, so subtract $P(A \text{ and } B)$. Disjoint events have no overlap; independent events satisfy $P(A \text{ and } B)=P(A)P(B)$. Two positive-probability disjoint events cannot be independent, since observing one rules out the other.

For without-replacement draws, update both the remaining favourable count and total at each step. For independent repeated choices the probabilities multiply without that update. State the sampling assumptions before calculating. A table of observed relative frequencies supports empirical probabilities for that population, not a guarantee about

every future draw.

Conditional probability restricts the denominator. Of 40 club members, 12 cycle; 8 of those cyclists wear a helmet. $P(H \mid C) = n(H \cap C)/n(C) = 8/12 = 2/3$. The group is cyclists, so 40 is not this denominator.

Original counts: condition on the requested group

Group	Lunch	No lunch	Total
Bus	18	12	30
Not bus	12	8	20
Total	30	20	50

Original worked example from existing native teaching; transfer tasks use their own data.

Worked example from the native lesson: Original counts: bus users–18 carry a packed lunch, 12 do not; non-bus users–12 carry lunch, 8 do not. Total 50. $P(\text{lunch})=30/50=0.6$; $P(\text{lunch given bus})=18/30=0.6$; $P(\text{bus given lunch})=18/30=0.6$ in this particular balanced table. Change one cell and these need not stay equal. $P(\text{bus and lunch})=18/50=0.36$. equal here to $0.6 \cdot 0.6$, so the table shows independence of these categories.

Exercises 2.1·2.2

2.1 Of 20 cyclists, 8 wear red. Of all 60 people, 18 wear red. $P(\text{red given cyclist})$: [1]

- A $8/20$
- B $8/60$
- C $20/60$

2.2 Two disjoint positive-probability events are independent. [1]

- A False
- B True
- C Always if totals match

Exercises 2.1·2.2

2.1 Of 20 cyclists, 8 wear red. Of all 60 people, 18 wear red. $P(\text{red given cyclist})$: [1]

- **A** $8/20$
- **B** $8/60$
- **C** $20/60$

2.1 A – The conditioning group is the 20 cyclists. B measures the joint share among everyone; C measures the share who cycle, ignoring red clothing.

Correct response linked to the given case.

2.2 Two disjoint positive-probability events are independent. [1]

- **A** False
- **B** True
- **C** Always if totals match

2.2 A – One event occurring makes the other impossible. B contradicts the zero intersection and positive product of event probabilities; C proposes a condition on totals that does not establish independence.

Correct response linked to the given case.

Exercises 2.3-3.1

2.3 $P(A)=0.35$. $P(\text{not } A)=$ _____. [1]

2.4 $P(A \text{ given } B)$ must equal $P(B \text{ given } A)$. [1]

3.1 A survey records 12 tea drinkers and 8 non-tea drinkers among 20 cyclists; among 30 walkers it records 9 tea drinkers and 21 non-tea drinkers. Find $P(\text{tea})$, $P(\text{tea} \mid \text{cyclist})$ and $P(\text{cyclist} \mid \text{tea})$. [3]

Exercises 2.3-3.1

2.3 $P(A)=0.35$. $P(\text{not } A)=$ _____. [1]

2.3 0.65 – Complement probabilities sum to 1.

Correct response linked to the given case.

2.4 $P(A \text{ given } B)$ must equal $P(B \text{ given } A)$. [1]

2.4 False – The denominator changes with the conditioning group.

Correct response linked to the given case.

3.1 A survey records 12 tea drinkers and 8 non-tea drinkers among 20 cyclists; among 30 walkers it records 9 tea drinkers and 21 non-tea drinkers. Find $P(\text{tea})$, $P(\text{tea} \mid \text{cyclist})$ and $P(\text{cyclist} \mid \text{tea})$. [3]

3.1 Total tea drinkers are 21 of 50, so $P(T) = 21/50$. Within cyclists, $P(T \mid C) = 12/20 = 3/5$. Within tea drinkers, $P(C \mid T) = 12/21 = 4/7$. These different denominators make the conditional probabilities unequal.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence

. Accurate conditions, checks or qualification.

Exercises 3.2-3.3

3.2 In the same survey, are tea drinking and cycling independent? Are they disjoint?
Use numbers for both answers. [3]

3.3 One respondent is chosen uniformly. Find the probability of walking or drinking tea, including those who do both. [3]

Exercises 3.2·3.3

3.2 In the same survey, are tea drinking and cycling independent? Are they disjoint? Use numbers for both answers. [3]

3.2 They are not independent because $P(T \mid C) = 3/5 \neq 21/50 = P(T)$. They are not disjoint because 12 respondents do both: $P(T \cap C) = 12/50 > 0$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 One respondent is chosen uniformly. Find the probability of walking or drinking tea, including those who do both. [3]

3.3 Use inclusion–exclusion. $P(W \cup T) = (30 + 21 - 9)/50 = 42/50 = 21/25$. The 9 tea-drinking walkers must not be counted twice. Equivalently exclude the 8 cyclists who do not drink tea.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.