

Exercise walk-through

10

AM-equivalence · Equivalent expressions and domain restrictions

SAT · SAT · SAT

10 Equivalent expressions and domain restrictions

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS equivalent expressions • factor

Objective

- Factor and expand expressions without changing their values
- Simplify rational and radical expressions on the original domain
- Use expression structure to identify zeros and coefficients

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

Expansion distributes each factor across the others; factorisation reverses that process. $x^2-5x+6=(x-2)(x-3)$, so the product form exposes zeros while the expanded form exposes coefficients. Choose the form that serves the question rather than treating one represen-

tation as always preferable.

Cancel factors, never terms in a sum. $(x^2-4)/(x-2)$ becomes $x+2$ only for $x \neq 2$, because the original expression is undefined at 2. A restriction may remain after every denominator disappears from the simplified formula. State it before cancelling, then carry it through comparisons or equation solving.

Exponent rules depend on their setting: $a^m a^n = a^{(m+n)}$ and $(a^m)^n = a^{(mn)}$ in the permitted real domain. Negative exponents form reciprocals and require nonzero bases. With real square roots, $\sqrt{(x^2)} = |x|$, because the principal square root is nonnegative; it is not x for every real x .

To check equivalence, a numerical substitution can expose a mistake, but one matching value does not prove two expressions equal. A valid algebraic identity plus matching domain does. When the question requests a coefficient, expand only as far as needed and

account for every contribution to that power.

Equivalent expressions must agree on the stated domain . $E(x) = (x^2 - 25)/(x - 5) = (x + 5)$ only for $x \neq 5$. Cancellation removes a factor, but it cannot restore an excluded input. Also $\sqrt{x^2} = |x|$, because the principal square root cannot be negative.

$$(x^2-25)/(x-5)$$



$x \neq 5$ before cancellation



$x+5$, still with $x \neq 5$

Original diagram of the worked relationship; read the full wording and qualifications.

Worked example from the native lesson: $(x^2-9)/(x-3)=(x-3)(x+3)/(x-3)=x+3$ for $x \neq 3$. At $x=3$ the original is undefined. For $x=-5$, $\sqrt{(x^2)}=5=|x|$. In $(2x+1)(x-4)$, the x coefficient is $-8+1=-7$ and the full expression is $2x^2-7x-4$.

Exercises 2.1·2.2

2.1 Simplify $(x^2-16)/(x-4)$, with $x \neq 4$.

[1]

- A $x+4$
- B $x-4$
- C x^2-4

2.2 For $x=-6$, $\sqrt{(x^2)}$ equals:

[1]

- A 6
- B -6
- C 36

Exercises 2.1·2.2

2.1 Simplify $(x^2-16)/(x-4)$, with $x \neq 4$.

[1]

- A $x+4$
- B $x-4$
- C x^2-4

2.1 A – Difference of squares gives $(x-4)(x+4)$. B loses the other factor $x+4$; C does not result from cancellation of whole factors.

Correct response linked to the given case.

2.2 For $x=-6$, $\sqrt{(x^2)}$ equals:

[1]

- A 6
- B -6
- C 36

2.2 A – The principal root is $|x|$. B violates the nonnegative principal-root convention; C is the squared value rather than its root.

Correct response linked to the given case.

Exercises 2.3-3.1

2.3 In $(x+2)(x+5)$, the x coefficient is _____. [1]

2.4 Matching at $x=1$ proves two expressions equivalent for all x . [1]

3.1 Simplify $F(x) = (x^2 - 4x)/(x^2 - 16)$. State every original restriction and explain why

Exercises 2.3·3.1

2.3 In $(x+2)(x+5)$, the x coefficient is _____. [1]

2.3 7 – The two x terms contribute $5x+2x$.

Correct response linked to the given case.

2.4 Matching at $x=1$ proves two expressions equivalent for all x . [1]

2.4 False – Different expressions can share one value.

Correct response linked to the given case.

3.1 Simplify $F(x) = (x^2 - 4x)/(x^2 - 16)$. State every original restriction and explain why

3.1 Factor numerator $x(x - 4)$ and denominator $(x - 4)(x + 4)$. The original domain excludes $x = 4, -4$. Cancellation gives $F(x) = x/(x + 4)$ for $x \neq 4, -4$. The latter formula alone permits 4, where it gives $1/2$, while $F(4)$ is undefined.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence
. Accurate conditions, checks or qualification.

Exercises 3.2·3.3

3.2 Find the coefficient of x in $(3x - 2)(x + 5)$ and evaluate $\sqrt{(x - 5)^2}$ when $x = 1$. [3]

3.3 A student tests $x = 2$ and concludes that x^2 and $2x$ are equivalent. Refute the

Exercises 3.2-3.3

3.2 Find the coefficient of x in $(3x - 2)(x + 5)$ and evaluate $\sqrt{(x - 5)^2}$ when $x = 1$. [3]

3.2 Expansion gives $3x^2 + 15x - 2x - 10 = 3x^2 + 13x - 10$, so the coefficient is 13. The root is $|x - 5|$; at $x = 1$, it is $|-4| = 4$, not -4 or 16.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

3.3 A student tests $x = 2$ and concludes that x^2 and $2x$ are equivalent. Refute the

3.3 At $x = 3$ the outputs are 9 and 6, so one matching test is not proof. Solve $x^2 = 2x$ as $x(x - 2) = 0$, giving only $x = 0, 2$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.