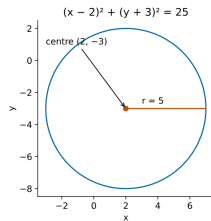


Circle equations, arcs and unit-circle trigonometry

Original skill preparation; versioned source scope

Method and conditions



- Complete both squares in a circle equation .
- Complete the square; radius; radians; right-triangle ratios.

Worked method 1

Read and reason

Complete both squares in a circle equation . $x^2 + y^2 + 6x - 8y = 0$ becomes $(x + 3)^2 + (y - 4)^2 = 25$. The centre is $(-3, 4)$ and radius is 5. Arc length is $s = r\theta$ only for radian angle θ .

Original context 1

Read and reason

Task context

Read and reason

Each task below states its own complete data. Values may differ from the worked example.

Attempt 1.1

Read and reason

Find the centre and radius of $x^2 + y^2 - 8x + 2y + 8 = 0$. Does $(7, -1)$ lie on it?

Reasoning 1.1

Read and reason

Complete squares to get $(x - 4)^2 + (y + 1)^2 = 9$. Centre $(4, -1)$, radius 3.
Substitution of $(7, -1)$ gives $3^2 + 0^2 = 9$, so it is on the circle.

Attempt 2.1

Read and reason

A sector has radius 8 cm and angle 135° . Find its angle in radians, arc length and area. An inscribed angle elsewhere on the circle intercepts this same arc; find it.

Reasoning 2.1

Read and reason

$\theta = 135\pi/180 = 3\pi/4$ radians. $s = r\theta = (8 \text{ cm})(3\pi/4) = 6\pi \text{ cm}$.

$A = r^2\theta/2 = (8 \text{ cm})^2(3\pi/4)/2 = 24\pi \text{ cm}^2$. The inscribed angle is half the central angle: 67.5 degrees.

Attempt 3.1

Read and reason

A unit-circle point is in quadrant III and has $y = -1/2$. Find its x -coordinate and one angle between 0° and 360° .

Reasoning 3.1

Read and reason

$x^2 + y^2 = 1$ gives $x^2 = 3/4$. Quadrant III requires $x < 0$, so $x = -\sqrt{3}/2$. The point is at 210 degrees, with reference angle 30 degrees.

Transfer and limits

Read and reason

Complete the square; radius; radians; right-triangle ratios.

Check yourself

Read and reason

Close the worked reasoning. Explain why the method fits the final transfer task, reconstruct its conclusion, and state a condition that would change your answer. Then check the full reasoning in the reference.