18 Scale factors, angle relationships and triangles

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS perpendicular height • scale factor • similarity • congruence

Objective

- Apply length, area and volume scaling to similar figures
- Use parallel-line angles and triangle sums to find unknowns
- Distinguish similarity from congruence and adequate information

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

For similar figures with length scale k, corresponding lengths scale by k, areas by k^2 and volumes by k^3. These laws apply when all corresponding dimensions change together. A cylinder with only radius doubled and fixed height has four times the volume, not eight times. State which dimensions are scaled before choosing the power.

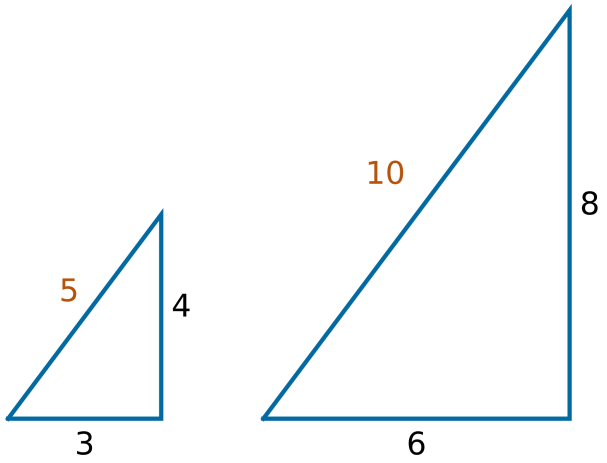
Use the correct geometric measure: triangle area is half base times perpendicular height ; cylinder volume is base area times height. Surface area adds exposed faces, while volume measures enclosed space. A slant length is not automatically the perpendicular height required by an area formula. Attach square or cubic units to the result.

Vertical angles are equal. For parallel lines cut by a transversal, corresponding and alternate interior angles are equal, and same-side interior angles sum to 180°. A triangle's interior angles sum to 180°; its exterior angle equals the sum of the two remote interior angles. These rules depend on stated or marked parallelism, not the drawing's appearance.

Similar triangles share corresponding angles and proportional corresponding sides; congruent triangles also have equal corresponding lengths. Two angles establish similarity but not size equality. Maintain correspondence when writing a ratio. A side–side–angle arrangement does not universally guarantee congruence, whereas side–angle–side includes the angle between the two sides.

A scale factor k changes every corresponding length. $A_2 = k^2 A_1$ and $V_2 = k^3 V_1$ for similar shapes. If $k = 3/2$ and $A_1 = 24\,\text{cm}^2$, $A_2 = (3/2)^2(24\,\text{cm}^2) = 54\,\text{cm}^2$. Changing only one dimension does not justify these similarity rules.

Length ×2; area ×4 for similar triangles



Doubling every length multiplies the area by 4

Worked example. Two similar solids have edge ratio 2:3, so surface-area ratio is 4:9 and volume ratio 8:27. A triangular garden with base 12 m and perpendicular height 5 m has area 30 m^2. If a triangle has interior angles 48° and 67°, its third is 65° and the adjacent exterior angle is 115°=48°+67°.

2 Practice

2.1 Similar solids have length scale 3. Volume scale: [1]

- **A** 27
- **B** 9
- **C** 3

2.2 Triangle angles 35° and 85° leave: [1]

- **A** 60°
- **B** 120°
- **C** 50°

2.3 Area scale for length scale 4 is _____. [1]

2.4 Two equal corresponding angles prove equal triangle sizes. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Similar containers have surface areas 96 and 216 square centimetres. The smaller volume is 160 cubic centimetres. Find the larger volume. [3]

3.2 A right triangle has hypotenuse 13 cm and one leg 5 cm. Find the other leg, its area, and the sine of the angle opposite the 5 cm leg. [3]

3.3 Two interior angles of a triangle are 42° and 73°. Find the third angle and its adjacent exterior angle. Would those angles alone determine all side lengths? [3]

3.4 Two parallel lines are cut by a transversal. One corresponding angle is 68°. Find its matching corresponding angle and the adjacent angle on the second line. Explain which given condition justifies each step. [3]