

14 Distributions, transformations and spread

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS standard deviation • frequency • median

Objective

- Compare mean, median, range and standard deviation qualitatively
- Predict the effect of adding or multiplying every data value
- Read frequency information without confusing values and counts

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

The mean uses the sum of all observations divided by their count; the median is the middle of the ordered observations. Frequency means a value repeats: if 2 occurs three times and 8 once, the mean is $(3 \cdot 2 + 8) / 4 = 3.5$, not $(2 + 8) / 2$. The horizontal positions on a display are values and the frequencies determine how many observations they represent.

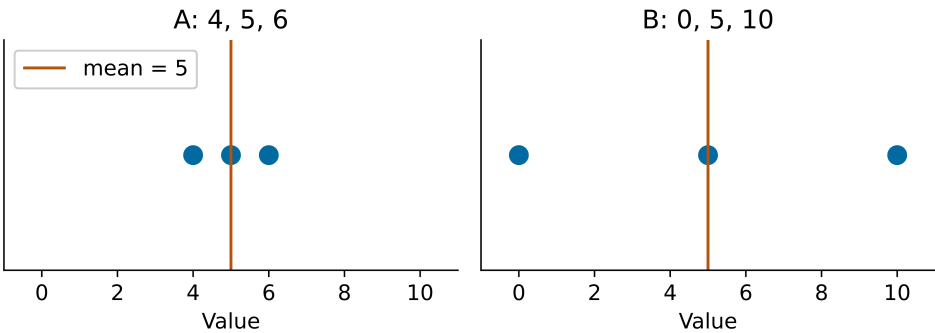
The range is maximum minus minimum; standard deviation describes typical distances from the mean. You need not memorise a full formula to compare simple equally centred distributions: data clustered close to the mean have smaller standard deviation than data moved further away symmetrically. An outlier can alter mean and spread substantially while leaving the median stable.

Adding c to every observation raises both mean and median by c but leaves range and standard deviation unchanged, because pairwise distances do not change. Multiplying every observation by a positive k multiplies mean, median, range and standard deviation by k . More generally spread scales by $|k|$, while a negative multiplier reverses ordering.

When comparing plots, inspect the scale and the number of observations before judging apparent width. A graph stretched horizontally can look more variable without representing different values. A claim about equality of standard deviations needs the actual distributions, not merely equal ranges or equal means.

A frequency counts repeated observations. Values 1, 3, 7 with frequencies 2, 1, 1 represent four observations: 1, 1, 3, 7. $\bar{x} = \sum fx / \sum f = (2(1) + 1(3) + 1(7)) / 4 = 3$. The median is $(1 + 3) / 2 = 2$; it need not equal the mean.

Same centre; different distances from the centre



Same mean, but B's values lie further from the centre, so B has the larger standard deviation

Worked example. $A = \{4, 5, 6\}$ and $B = \{0, 5, 10\}$ both have mean and median 5, but B has greater spread. Adding 7 gives $A + 7 = \{11, 12, 13\}$, mean 12 and the same standard deviation as A. Multiplying A by 2 gives $\{8, 10, 12\}$; its mean is 10 and its range and standard deviation double. Equal range alone does not guarantee equal standard deviation.

2 Practice

2.1 Add 10 to every observation. Standard deviation: [1]

- **A** Stays the same
- **B** Increases by 10
- **C** Doubles

2.2 2 occurs three times; 6 occurs once. Mean: [1]

- **A** 3
- **B** 4
- **C** 8

2.3 The range of 3, 7, 9 is _____. [1]

2.4 Two distributions with equal means must have equal spread. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Values 2, 5, 8 have frequencies 3, 2, 1. Find mean, median and range. [3]

3.2 Every observation above is changed by $z = 3x - 4$. Find the new mean, median and range. State how standard deviation changes without computing it. [3]

3.3 Compare $A = \{0, 0, 10, 10\}$ and $B = \{0, 5, 5, 10\}$. Both have range 10. Must their standard deviations agree? Justify. [3]
