

12 Nonlinear functions, transformations and growth

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS transformation • vertex form • parabola • rate

Objective

- Interpret zeros, vertices and parameters from useful function forms
- Distinguish additive linear change from multiplicative exponential change
- Transform a graph while tracking its input and output

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

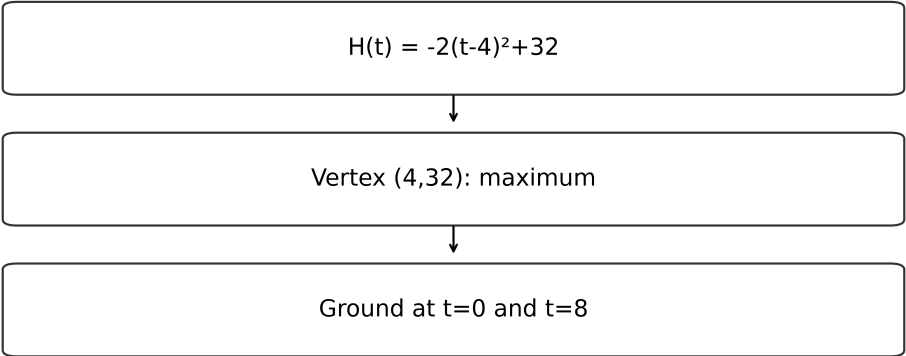
Vertex form $a(x-h)^2+k$ identifies the vertex (h,k) and opening direction; factored form identifies zeros. The same function can have both forms. In context, determine which features are meaningful: a maximum height occurs at the vertex of a downward parabola, while a zero can represent returning to ground level.

For $y=A b^t$, A is the value at $t=0$ and b is the factor per unit time. A growth rate r gives $b=1+r$; decay gives $b=1-r$. Equal differences indicate a linear model, while equal ratios indicate an exponential model for equal input steps. Neither model should be extrapolated beyond a justified domain without a contextual reason.

An output shift $f(x)+k$ moves every point vertically by k . An input shift $f(x-h)$ moves the graph right by h : the same old input is reached when the new x is larger. A multiplier outside scales output; a negative outside reflects across the x -axis. Determine the transformation algebraically instead of memorising a direction without checking a point.

For an exponential factor given over several time units, distinguish that interval from one unit. Doubling every three hours means $2^{t/3}$, not 2^t . For a quadratic, use the requested representation to read an extremum or solve for an input. Keep initial values, growth factors and growth percentages separate.

Vertex form $a(x-h)^2+k$ reveals the turning point (h,k) . For $H(t)=-2(t-4)^2+32$, the maximum is 32 when $t=4$. Setting $H(t)=0$ gives $(t-4)^2=16$, hence $t=0,8$. Restrict time to the physical interval when the problem represents a flight.



Worked steps: $H(t) = -2(t-4)^2+32 \rightarrow \text{Vertex } (4,32): \text{maximum} \rightarrow \text{Ground at } t=0 \text{ and } t=8$

Worked example. $h(t)=-4(t-3)^2+36$ has a maximum 36 at $t=3$, with zeros $t=0$ and 6. $N(t)=80(1.25)^t$ starts at 80 and grows 25% per unit; $N(2)=125$. A quantity doubling every three hours is $80 \cdot 2^{t/3}$, so after six hours it is 320. If $f(x)=x^2$, $f(x-2)+5$ has vertex $(2,5)$.

2 Practice

2.1 In $200(0.8)^t$, the decay rate per unit is: [1]

- A 20%
- B 80%
- C 200%

2.2 The vertex of $(x+3)^2-7$ is: [1]

- A (-3,-7)
- B (3,-7)
- C (-3,7)

2.3 100 grows by 10% twice. The result is _____. [1]

2.4 Equal additive differences prove exponential growth. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 A model height is $H(t) = -3(t - 2)^2 + 12$ metres, with t seconds from launch until return to ground. Find the domain, greatest height, and time interval with height at least 9 metres. [3]

3.2 A culture model starts with 120 cells and grows by a factor of 1.5 every two hours. Write $N(t)$ for hours t , find $N(4)$, and distinguish factor from percentage growth. [3]

3.3 If $f(x) = (x + 1)^2$, find the vertex of $g(x) = -2f(x - 3) + 5$ and its maximum or minimum. [3]
