

9 Linear parameters, solution counts and inequalities

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS identity • rate

Objective

- Solve and classify one-variable linear equations with parameters
- Interpret two-variable linear equations as sets of ordered pairs
- Interpret slope and intercept of a contextual linear function
- Classify two-equation linear systems and check their intersections
- Represent strict and inclusive inequalities in one or two variables

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

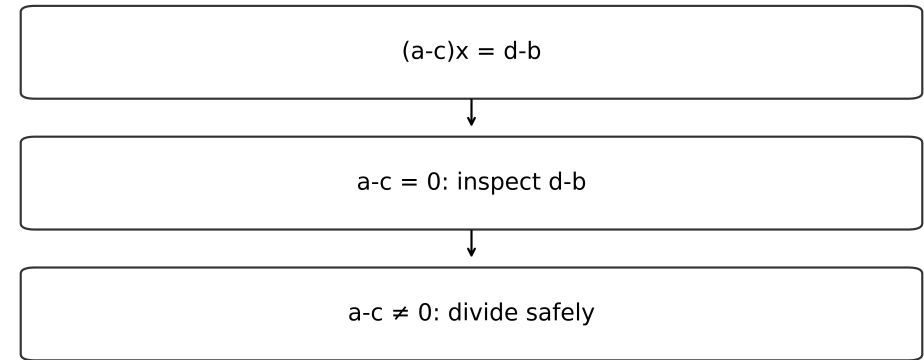
Collect variable terms and constants separately. An equation $ax+b=cx+d$ becomes $(a-c)x=d-b$. If $a-c$ is nonzero there is one solution; if both sides reduce to zero there are infinitely many; if the variable coefficient is zero but the constant is nonzero there are none. Never divide by a parameter before checking whether it can be zero.

A two-variable equation gives a set of ordered pairs. In $y=mx+b$, m is the change in y per unit x and b is y at $x=0$. For a contextual model, attach units to both: a taxi charge $C=3t+5$ has a 5-unit initial fee and 3 units per time unit. The intercept may be outside the practical domain even when it exists algebraically.

For a system, compare both coefficients and constants. Multiplying one whole equation by a nonzero number produces the same line; matching variable coefficients with different constants gives parallel distinct lines. Otherwise an intersection supplies the pair satisfying both equations. Substitute the pair into both original equations to check it.

Multiplying or dividing an inequality by a negative number reverses its direction. A strict inequality excludes the boundary, while $\leq$ or $\geq$ includes it. For two variables, draw the corresponding line and test a point away from it to identify the allowed half-plane. The diagram is a representation of all solutions, not a single preferred point.

An identity _____ holds for every permitted input. Start with $ax+b=cx+d$. Collect terms: $(a-c)x=d-b$. Inspect $a-c$ before dividing. If $a=c$ and $b=d$, every real input works. If only $a=c$, no input works. Otherwise $x=(d-b)/(a-c)$ gives one solution.



Worked steps: $(a-c)x = d-b \rightarrow a-c = 0$: inspect $d-b \rightarrow a-c \neq 0$: divide safely

Worked example. For $kx+6=2x+6$, $k=2$ gives $0=0$ and infinitely many solutions; $k \neq 2$ gives $x=0$. For $-2x+3 < 11$, subtract 3 and divide by -2 to get $x > -4$. For $y \leq 2x+1$, $(0,0)$ works and the solid boundary is included. The system $2x+4y=10$ and $x+2y=6$ has no solution because doubling the second gives the same left side equal to 12, not 10.

2 Practice

2.1 For $ax+4=3x+4$, which a gives infinitely many solutions? [1]

- A 3
- B 0
- C 4

2.2 Solve $-3x \leq 12$. [1]

- A $x \geq -4$
- B $x \leq -4$
- C $x \geq 4$

2.3 For $y=7x+9$, the y -intercept is _____. [1]

2.4 The system $x+y=2$ and $2x+2y=5$ has an intersection. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Classify the solutions of $(p-1)x+3=2x+p$ for every real p . [3]

3.2 A hire charge C yuan has a fixed fee f yuan and rate r yuan per hour. Three hours cost 31 yuan; seven hours cost 59 yuan. Find f, r and the greatest affordable whole number of hours with 80 yuan. [3]

3.3 Describe all points satisfying $y > 2x - 1$ and $y \leq -x + 5$. Is $(2, 3)$ included? State the possible x -values. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 A – At $a=3$ both sides are the same expression. B gives $-3x=0$ and only $x=0$; C gives $x=0$. Neither is an identity.

Correct response linked to the given case.

2.2 A – Dividing by -3 reverses $\leq$ to $\geq$. B forgets to reverse the inequality after division by a negative; C changes the sign of the boundary incorrectly.

Correct response linked to the given case.

2.3 9 – Set $x=0$ to obtain $y=9$.

Correct response linked to the given case.

2.4 False – Doubling the first gives $2x+2y=4$, inconsistent with 5.

Correct response linked to the given case.

3.1 Collect terms to get $(p-3)x = p-3$. At $p=3$, the equation is $0=0$, so every real x works. At $p \neq 3$, divide by $p-3$ to get $x=1$. There is no value of p giving no solution. Substitution of $x=1$ gives $p+2$ on both sides.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence
. Accurate conditions, checks or qualification.

3.2 Use $C = f + rt$. The equations are $31 = f + r(3\text{ h})$ and $59 = f + r(7\text{ h})$. Subtract: $r = (59 - 31)\text{ yuan}/(7 - 3)\text{ h} = 7\text{ yuan/h}$. Then $f = 31\text{ yuan} - (7\text{ yuan/h})(3\text{ h}) = 10\text{ yuan}$. Budget: $10 + 7t \leq 80$, so $t \leq 10$ hours. Ten whole hours is affordable; eleven costs 87 yuan.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.

3.3 The first boundary is dashed and the second solid. The common region is above the first line and on or below the second. At $(2, 3)$ the first condition is $3 > 3$, which fails. For the vertical interval to exist, $2x - 1 < -x + 5$, hence $x < 2$. The lines meet at $(2, 3)$ but their meeting point is excluded.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence.
Accurate conditions, checks or qualification.