19 Circle equations, arcs and unit-circle trigonometry

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS circle equation • arc length • tangent • radian

Objective

- Recover circle centre and radius by completing the square
- Convert central angles and calculate arc length and sector area
- Use tangent , inscribed-angle and unit-circle relationships

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

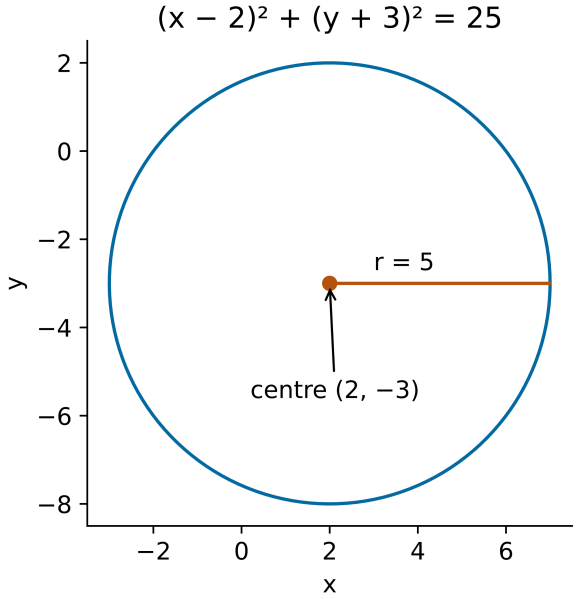
A circle with centre (h,k) and radius r satisfies $(x-h)^2+(y-k)^2=r^2$. For an expanded equation, complete the square in x and y and add the balancing constants to the other side. The resulting right side must be positive for a real nondegenerate circle. Keep the radius distinct from its square.

A central angle θ in radians gives arc length $r\theta$ and sector area $r^2\theta/2$. In degrees, use the fraction $\theta/360$ of the full circumference or area. A full turn is 2π radians=360°. The formulas $r\theta$ and $r^2\theta/2$ require radians; inserting degrees without conversion gives a wrong scale.

A tangent at a circle point is perpendicular to the radius there. An inscribed angle is half the central angle subtending the same arc; an angle subtending a diameter is a right angle. Identify the actual shared arc and centre before applying these statements. A chord is not a tangent and need not be perpendicular to a radius unless further conditions establish it.

On the unit circle, a point at angle θ has coordinates $(\cos\theta, \sin\theta)$, so $\cos^2\theta+\sin^2\theta=1$. Signs depend on quadrant. For acute complementary angles, $\sin\theta=\cos(90°-\theta)$. Use special triangles for exact values: at 30° the sine is 1/2 and cosine $\sqrt{3}/2$; at 45° both are $\sqrt{2}/2$.

Complete both squares in a circle equation . $x^2+y^2+6x-8y=0$ becomes $(x+3)^2+(y-4)^2=25$. The centre is $(-3,4)$ and radius is 5. Arc length is $s=r\theta$ only for radian angle θ.



Read the centre from the signs inside the brackets, and the radius as $\sqrt{25}=5$

Worked example. $x^2+y^2-4x+6y-12=0$ becomes $(x-2)^2+(y+3)^2=25$, giving centre (2,-3), radius 5. For radius 6 and central angle $\pi/3$, arc length is 2π and sector area 6π. An inscribed angle on that same 60° arc is 30°. At 150°, cosine is $-\sqrt{3}/2$ and sine 1/2 because the point is in quadrant II.

2 Practice

2.1 Centre and radius of $(x+1)^2+(y-4)^2=9$: [1]

- **A** (-1,4), 3
- **B** (1,-4), 9
- **C** (-1,4), 9

2.2 Radius 4 with central angle $\pi/2$ gives arc length: [1]

- **A** 2π
- **B** 8π
- **C** $\pi/8$

2.3 180 degrees equals _____ radians. [1]

2.4 The angle between a tangent and its radius at contact is 90 degrees. [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 Find the centre and radius of $x^2 + y^2 - 8x + 2y + 8 = 0$. Does $(7, -1)$ lie on it? [3]

3.2 A sector has radius 8 cm and angle 135°. Find its angle in radians, arc length and area. An inscribed angle elsewhere on the circle intercepts this same arc; find it. [3]

3.3 A unit-circle point is in quadrant III and has $y = -1/2$. Find its x-coordinate and one angle between 0° and 360°. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 A – The centre signs follow x-h,y-k and radius is $\sqrt{9}$. B reverses both centre signs and uses radius squared; C gets the centre right but retains the squared radius.

Correct response linked to the given case.

2.2 A – rθ=$4 \cdot \pi/2$=2π. B fails to halve 4π; C divides the angle by radius rather than multiplying.

Correct response linked to the given case.

2.3 π – A half-turn is π radians.

Correct response linked to the given case.

2.4 True – The tangent-radius perpendicular theorem applies at the contact point.

Correct response linked to the given case.

3.1 Complete squares to get $(x-4)^2+(y+1)^2 = 9$. Centre $(4, -1)$, radius 3. Substitution of $(7, -1)$ gives $3^2 + 0^2 = 9$, so it is on the circle.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence . Accurate conditions, checks or qualification.

3.2 $\theta = 135\pi/180 = 3\pi/4$ radians. $s = r\theta = (8\,\text{cm})(3\pi/4) = 6\pi\,\text{cm}$. $A = r^2\theta/2 = (8\,\text{cm})^2(3\pi/4)/2 = 24\pi\,\text{cm}^2$. The inscribed angle is half the central angle: 67.5 degrees.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 $x^2 + y^2 = 1$ gives $x^2 = 3/4$. Quadrant III requires $x < 0$, so $x = -\sqrt{3}/2$. The point is at 210 degrees, with reference angle 30 degrees.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.