SAT • EXERCISE SHEET

16 Probability from a table and a restricted group

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS conditional probability • independent events

Objective

- Calculate event and complement probabilities from counts
- Use the conditioning group as the denominator
- Distinguish independence from disjointness

Original public practice; marks below are classroom self-check points, not official criteria.

1 Key knowledge and worked examples

A probability based on equally represented observations is favourable count divided by the relevant total. A two-way table has joint counts in cells, group totals on the margins and a grand total. Read whether the question asks about a joint event, either event or one event given another before choosing the denominator.

For P(A given B), restrict attention to the B group and divide the A-and-B count by the B total. It is not generally equal to P(B given A), because the conditioning groups differ. A fraction with the correct numerator but the wrong population still answers the wrong question.

The complement probability is 1-P(A). For either A or B, adding separate probabilities double-counts overlap, so subtract P(A and B). Disjoint events have no overlap; independent events satisfy P(A and B)=P(A)P(B). Two positive-probability disjoint events cannot be independent, since observing one rules out the other.

For without-replacement draws, update both the remaining favourable count and total at each step. For independent repeated choices the probabilities multiply without that update. State the sampling assumptions before calculating. A table of observed relative frequencies supports empirical probabilities for that population, not a guarantee about every future draw.

Conditional probability restricts the denominator. Of 40 club members, 12 cycle; 8 of those cyclists wear a helmet. $P(H \mid C) = n(H \cap C)/n(C) = 8/12 = 2/3$. The group is cyclists, so 40 is not this denominator.

Original counts: condition on the requested group

Group	Lunch	No lunch	Total
Bus	18	12	30
Not bus	12	8	20
Total	30	20	50

For a conditional probability, divide by the total of the requested group only

Worked example. Original counts: bus users–18 carry a packed lunch, 12 do not; non-bus users–12 carry lunch, 8 do not. Total 50. P(lunch)=30/50=0.6; P(lunch given bus)=18/30=0.6; P(bus given lunch)=18/30=0.6 in this particular balanced table. Change one cell and these need not stay equal. P(bus and lunch)=18/50=0.36, equal here to 0.6 · 0.6, so the table shows independence of these categories.

2 Practice

2.1 Of 20 cyclists, 8 wear red. Of all 60 people, 18 wear red. P(red given cyclist): [1]

- **A** 8/20
- **B** 8/60
- **C** 20/60

2.2 Two disjoint positive-probability events are independent. [1]

- **A** False
- **B** True
- **C** Always if totals match

2.3 P(A)=0.35. P(not A)=_____. [1]

2.4 P(A given B) must equal P(B given A). [1]

3 Exam-style questions

Use the data stated in each original task.

3.1 A survey records 12 tea drinkers and 8 non-tea drinkers among 20 cyclists; among 30 walkers it records 9 tea drinkers and 21 non-tea drinkers. Find $P(tea)$, $P(tea \mid cyclist)$ and $P(cyclist \mid tea)$. [3]

3.2 In the same survey, are tea drinking and cycling independent? Are they disjoint? Use numbers for both answers. [3]

3.3 One respondent is chosen uniformly. Find the probability of walking or drinking tea, including those who do both. [3]

Solutions

Accept equivalent correct methods and supported interpretations. The marks check reasoning; they are not an official grade.

2.1 A – The conditioning group is the 20 cyclists. B measures the joint share among everyone; C measures the share who cycle, ignoring red clothing.

Correct response linked to the given case.

2.2 A – One event occurring makes the other impossible. B contradicts the zero intersection and positive product of event probabilities; C proposes a condition on totals that does not establish independence.

Correct response linked to the given case.

2.3 0.65 – Complement probabilities sum to 1.

Correct response linked to the given case.

2.4 False – The denominator changes with the conditioning group.

Correct response linked to the given case.

3.1 Total tea drinkers are 21 of 50, so $P(T) = 21/50$. Within cyclists, $P(T \mid C) = 12/20 = 3/5$. Within tea drinkers, $P(C \mid T) = 12/21 = 4/7$. These different denominators make the conditional probabilities unequal.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence . Accurate conditions, checks or qualification.

3.2 They are not independent because $P(T \mid C) = 3/5 \neq 21/50 = P(T)$. They are not disjoint because 12 respondents do both: $P(T \cap C) = 12/50 > 0$.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.

3.3 Use inclusion–exclusion. $P(W \cup T) = (30 + 21 - 9)/50 = 42/50 = 21/25$. The 9 tea-drinking walkers must not be counted twice. Equivalently exclude the 8 cyclists who do not drink tea.

Sound conclusion or proposed method. Relevant supporting steps or textual evidence. Accurate conditions, checks or qualification.